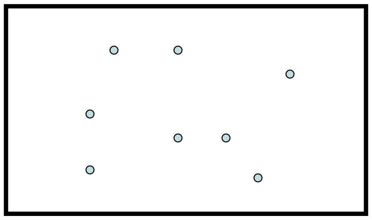

Monatomic ideal gas

(1) Only translational kinetic energy

(2) No potential energy

$\Rightarrow$ Total energy $E = \frac{1}{2}mv^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$

$g(v_x)dv_x$ Is the probability that the x-component of the velocity of a particle is between v_x and $v_x + dv_x$

The energy associated with a velocity component v_x is = $\frac{1}{2}mv_x^2$

Since $P(\varepsilon) \propto e^{-\frac{\varepsilon}{k_B T}}$

$$\Rightarrow g(v_x)dv_x \propto e^{-\frac{mv_x^2}{2k_B T}} dv_x$$

$$g(v_x)dv_x \propto e^{-\frac{mv_x^2}{2k_B T}} dv_x$$

$$\Rightarrow g(v_x)dv_x = C_1 e^{-\frac{mv_x^2}{2k_B T}} dv_x \longrightarrow (1)$$

$$\int_{-\infty}^{\infty} g(v_x)dv_x = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} C_1 e^{-\frac{mv_x^2}{2k_B T}} dv_x = 1 \longrightarrow (2)$$

Comparing eqn. 2 with eqn. 3

$$\int_{-\infty}^{\infty} x^{2n} e^{-\alpha x^2} dx = \frac{(2n)!}{n! 2^{2n}} \sqrt{\frac{\pi}{\alpha^{2n+1}}} \longrightarrow (3)$$

$$\int_{0}^{\infty} x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}} \longrightarrow (4)$$

$n = 0$

$\alpha = \frac{m}{2k_B T}$

$$\Rightarrow C_1 \frac{0!}{0! 2^0} \sqrt{\frac{\pi}{(m/2k_B T)^1}} = 1$$

Using this value for C_1 in eqn. (1) we get:

$$\Rightarrow g(v_x)dv_x = \sqrt{\frac{m}{2\pi k_B T}} e^{-\frac{mv_x^2}{2k_B T}} dv_x$$

Since $\frac{1}{2}mv^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$

$$g(v_x)dv_x = C_1 e^{-\frac{mv_x^2}{2k_BT}} dv_x, \quad g(v_y)dv_y = C_1 e^{-\frac{mv_y^2}{2k_BT}} dv_y, \quad g(v_z)dv_z = C_1 e^{-\frac{mv_z^2}{2k_BT}} dv_z$$

$\Rightarrow$ Probability of velocity being between $\vec{v}$ and $\vec{v} + d\vec{v}$ is:

$$f'(v_x, v_y, v_z)dv_x dv_y dv_z = g(v_x)g(v_y)g(v_z)dv_x dv_y dv_z = C_1^3 e^{-\frac{mv_x^2}{2k_BT}} e^{-\frac{mv_y^2}{2k_BT}} e^{-\frac{mv_z^2}{2k_BT}} dv_x dv_y dv_z,$$

$$\Rightarrow f'(v_x, v_y, v_z)dv_x dv_y dv_z = C_1^3 e^{-\frac{mv_x^2}{2k_BT} - \frac{mv_y^2}{2k_BT} - \frac{mv_z^2}{2k_BT}} dv_x dv_y dv_z,$$

$$\Rightarrow f'(v_x, v_y, v_z)dv_x dv_y dv_z = C_1^3 e^{-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2k_BT}} dv_x dv_y dv_z,$$

$$\Rightarrow f'(v_x, v_y, v_z)dv_x dv_y dv_z = C_1^3 e^{-\frac{mv^2}{2k_BT}} dv_x dv_y dv_z,$$

In Cartesian coordinates: $dV = dxdydz$

In spherical coordinates: $dV = r^2 \sin\theta dr d\theta d\phi$

$$\Rightarrow f'(v_x, v_y, v_z)dv_x dv_y dv_z = C_1^3 e^{-\frac{mv^2}{2k_BT}} dv_x dv_y dv_z = C_1^3 e^{-\frac{mv^2}{2k_BT}} v^2 \sin\theta dv d\theta d\phi$$

Integrating over θ and ϕ gets rid of the direction information i.e. instead of the velocity distribution you get the SPEED distribution:

$f(v)dv$ is the probability that the speed of a particle in an ideal gas is between v and $v + dv$

$$\Rightarrow f(v)dv = \iint_{\theta,\phi} C_1^3 e^{-\frac{mv^2}{2k_BT}} v^2 \sin\theta dv d\theta d\phi$$

$$\Rightarrow f(v)dv = C_1^3 e^{-\frac{mv^2}{2k_BT}} v^2 dv \int_{\theta=0}^{\pi} \sin\theta d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$\Rightarrow f(v)dv = C_1^3 e^{-\frac{mv^2}{2k_BT}} v^2 dv \int_{\theta=0}^{\pi} \sin\theta d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$\Rightarrow f(v)dv = C_1^3 e^{-\frac{mv^2}{2k_BT}} v^2 dv.(-\cos\theta|_0^{\pi}).(\phi|_0^{2\pi})$$

$$\Rightarrow f(v)dv = C_1^3 e^{-\frac{mv^2}{2k_BT}} v^2 dv.2.2\pi$$

Putting all the constants together:

$$\Rightarrow f(v)dv = C_2 e^{-\frac{mv^2}{2k_BT}} v^2 dv$$

For a normalized *f(v)*:

$$\int_0^{\infty} f(v)dv = 1$$

Limits of $v = 0$ to ∞, because v is speed.

$$\int_0^{\infty} f(v)dv = 1$$

$$\Rightarrow \int_0^{\infty} f(v)dv = \int_0^{\infty} C_2 e^{-\frac{mv^2}{2k_BT}} v^2 dv = 1 \longrightarrow (5)$$

Comparing eqn. 5 with eqn. 3a

$$n = 1$$

$$\alpha = \frac{m}{2k_BT}$$

$$\int_{-\infty}^{\infty} x^{2n} e^{-\alpha x^2} dx = \frac{(2n)!}{n! 2^{2n}} \sqrt{\frac{\pi}{\alpha^{2n+1}}} \longrightarrow (3)$$

Since integrand is even:

$$2\int_0^{\infty} x^{2n} e^{-\alpha x^2} dx = \frac{(2n)!}{n! 2^{2n}} \sqrt{\frac{\pi}{\alpha^{2n+1}}}$$

$$\Rightarrow \int_0^{\infty} x^{2n} e^{-\alpha x^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{\alpha^{2n+1}}} \longrightarrow (3a)$$

$$\int_0^{\infty} x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}} \longrightarrow (4)$$

$$\Rightarrow \int_0^{\infty} C_2 e^{-\frac{mv^2}{2k_BT}} v^2 dv = C_2 \frac{2!}{1! 2^3} \sqrt{\frac{\pi}{(m/2k_BT)^3}} = 1$$

$$\Rightarrow C_2 = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_BT}\right)^{3/2}$$

$f(v)dv$	is the probability that the speed of a particle in an ideal gas is between v and $v + dv$

$$f(v)dv = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2} v^2 e^{\frac{-mv^2}{2k_BT}} dv$$

Maxwell-Boltzmann **speed** distribution

Maxima commands

```
(%i1) vmax:5;
(%o1) 5

(%i2) plot2d((v^2)*exp(-((v/vmax)^2)),[v,0,3*vmax]);
```