PHY 3101 "Introduction to Modern Physics" Fall 2011 16 September 2011
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Problem 2.21 from the textbook: (recall drawing from class), where the notation is $m_{o,p} = 938$ MeV/c^2 is the rest mass of the proton, $m_{o,\pi} = 135$ MeV/c^2 is the rest mass of the π^0, the neutral pion, and "in" will mean "incident" for the proton that is moving in the lab frame prior to the collision. Recall

$$E^2 = p^2c^2 + (m_o c^2)^2 \quad , \tag{1}$$

$$E = mc^2 \quad , \tag{2}$$

and the invariant is

$$(m_o c^2)^2 = E^2 - p^2c^2 \quad . \tag{3}$$

So, the lab frame invariant before collision is the left-hand-side (LHS) and the center-of-mass (CM) frame immediately after collision at threshold is the right-hand-side (RHS) of an equation that sets the invariants to be equal, $i.e.$

$$E_{lab}^2 - p_{lab}^2 c^2 = E_{CM}^2 - p_{CM}^2 c^2 \tag{4}$$

$$\left\{m_{in}c^2 + m_{o,p}c^2\right\}^2 - p_{in}^2c^2 = \left(2m_{o,p}c^2 + m_{o,\pi}c^2\right)^2 \quad , \tag{5}$$

where we need to keep in mind that m_{in} is the relativistic mass used in Eq. (2) and where we have used the fact that there is no momentum in the CM frame at threshold so any "p" terms from Eq. (3) are zero. We can now manipulate this result to obtain

$$m_{in}^2c^4 + 2m_{in}c^2m_{o,p}c^2 + m_{o,p}^2c^4 - p_{in}^2c^2 = 4m_{o,p}^2c^4 + 4m_{o,p}c^2m_{o,\pi}c^2 + m_{o,\pi}^2c^4 \quad , \tag{6}$$

but recognize that the first and last terms on the LHS are

$$m_{in}^2c^4 - p_{in}^2c^2 = m_{o,p}^2c^4 \quad , \tag{7}$$

so Eq. (6) becomes

$$2m_{in}c^2m_{o,p}c^2 + 2m_{o,p}^2c^4 = 4m_{o,p}^2c^4 + 4m_{o,p}c^2m_{o,\pi}c^2 + m_{o,\pi}^2c^4 \quad . \tag{8}$$

Now we simplify to get

$$m_{in}c^2m_{o,p}c^2 + m_{o,p}^2c^4 = 2m_{o,p}^2c^4 + 2m_{o,p}c^2m_{o,\pi}c^2 + \frac{1}{2}m_{o,\pi}^2c^4 \quad , \tag{9}$$

$$m_{in}c^2m_{o,p}c^2 = m_{o,p}^2c^4 + 2m_{o,p}c^2m_{o,\pi}c^2 + \frac{1}{2}m_{o,\pi}^2c^4 \quad , \tag{10}$$

$$m_{in}c^2 = m_{o,p}c^2 + 2m_{o,\pi}c^2 + \frac{1}{2}\frac{m_{o,\pi}^2c^4}{m_{o,p}c^2} \quad . \tag{11}$$

Now we recognize that the energy just beyond the rest mass energy is the threshold energy, E_t, which is the last two terms on the RHS of Eq. (11). So,

$$E_t = 2m_{o,\pi}c^2 + \frac{1}{2}\frac{m_{o,\pi}^2c^4}{m_{o,p}c^2} = m_{o,\pi}c^2\{2 + \frac{1}{2}\frac{m_{o,\pi}c^2}{m_{o,p}c^2}\} \quad . \tag{12}$$

Putting in the vaules we get $E_t = 280$ MeV. The MOST IMPORTANT POINT is that this threshold energy is more that twice the rest mass energy of the pion, as can quickly be seen from Eq. (12). Furthermore, if we made the approximation that $(m_{o,\pi}c^2/m_{o,p}c^2) \ll 1$, then we would not have gotten this "extra-energy required", $i.e.$ the last term in Eq. (12) would have been neglected (but it is a VERY important term!). Finally, if we were looking for the TOTAL energy of the protons in the beam that were incident on the fixed target, then this energy would be (938 + 280) MeV = 1217 MeV.