PHY4324–Electromagnetism II

Fall 2011

Test 2 – 55 minutes

Nov. 7, 2011

No other materials except calculators and one review sheet allowed. If you can't do one part of a problem, solve subsequent parts in terms of unknown answer–define clearly. **Do 2 of 4 problems, CLEARLY indicating which you want graded by circling the problem number!**. *Each problem is worth 15 pts., for maximum of 30 points.*

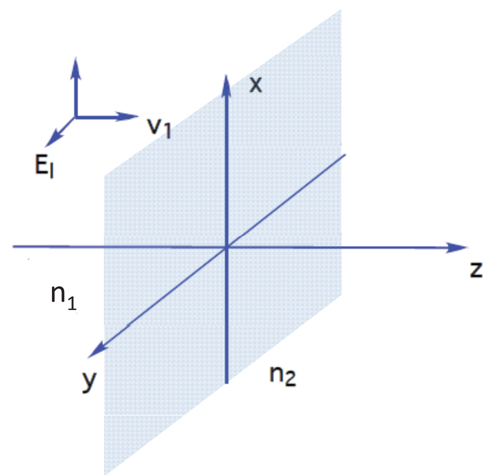

Figure 1: Plane wave in medium 1 traveling along z incident from left on interface to medium 2.

1. **Plane wave.** Suppose the xy-plane forms the boundary between two linear media with indices n_1 (take $\mu = \mu_0$ for both media), n_2. A plane wave $\mathbf{E}_I(z,t) = \mathbf{E_0} e^{ik_1 z - \omega t}\hat{y}$ traveling in the z direction and polarized in the $\hat{y}$ direction approaches the interface from the left (figure above).

   (a) (4 pts.) Calculate $\mathbf{B}_I(z,t)$ and indicate its direction in the figure.

   (b) (4 pts.) Write down the form of the reflected electric and magnetic field, $\mathbf{E}_R(z,t)$, $\mathbf{B}_R(z,t)$ and the transmitted electric and magnetic fields, $\mathbf{E}_T(z,t)$, $\mathbf{B}_T(z,t)$ respectively.

(c) (4 pts.) Write down boundary conditions for the fields at z=0 and use them to calculate the reflected and transmitted **E** fields in terms of incident field, indicating the directions of these fields in the figure.

(d) (3 pts.) If medium 2 is a conductor with conductivity σ and permeability μ_0, permittivity ϵ instead of a dielectric, what is the new form of the tranmitted electric field? (For full credit give the mathematical form, the amplitude of the electric field in region 2 in terms of the incident amplitude, and the dispersion relation.)

2. **Parallel-plate waveguide.** A waveguide consists of two infinite parallel plates, as shown. Consider TM waves propagating in the $+z$ direction, and assume that the fields are *uniform* along the x direction.

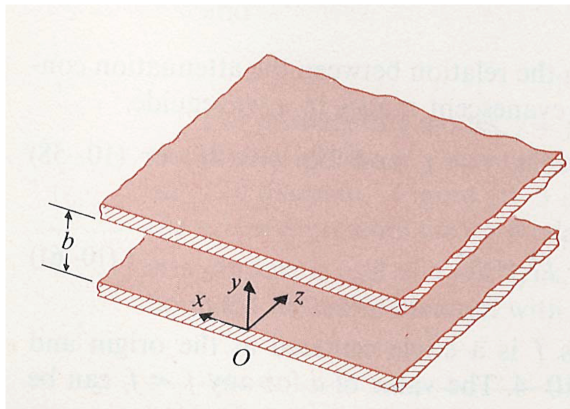

Figure 2: Infinite parallel-plate waveguide

(a) (4 pts.) State the boundary conditions on the electric and magnetic fields at the inner surfaces of the plates.

(b) (4 pts.) Write the equation for the z component of the electric field amplitude $E_z^0(y)$, and state the solution, consistent with the boundary conditions you found in (a). (*Hint:* even if you don't know the equation, you should be able to guess the solution.)

(c) (4 pts.) Using Maxwell's equations, find the transverse components $B_x^0(y)$ and $E_y^0(y)$.

(d) (3 pts.) Find the dispersion of the TM modes $\mathbf{k}(\omega)$ and state the cutoff frequency.

3. **Point charge.** The electric field of a moving point charge q moving at constant velocity $v\hat{x}$ has the form

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\frac{1-(v/c)^2}{(1-(v/c)^2\sin^2\theta)^{3/2}}\hat{R}, \tag{1}$$

where θ is the angle between the vector $\mathbf{r}$ and the x axis, and $\mathbf{R}$ is the vector from the particle to the field point $\mathbf{r}$ at the present time.

(a) (4 pts.) Sketch the electric field lines for the case v close to c.

(b) (4 pts.) Find the magnetic field.

(c) (4 pts.) Use the result (1) to derive the expression for the electric field of an infinite line charge: assume there are infinitely many charge elements $dq = \lambda dx$, each moving with velocity v, and integrate the charge densities to find the total electric field a distance s from the axis. *Hint:* You may need $\int_0^\pi d\theta \sin\theta/(1 - b^2 \sin^2\theta)$.

(d) (3 pts.) Go back to pt. charge moving with velocity v. At $t = 0$ the charge is located at the origin $x = 0$. How much energy from the point charge is passing through the plane $\perp$ to the x-axis at $x = a$? Set up in terms of a θ-integral which you need not evaluate. *Hint:* you may need $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$.

4. **Wire loop.**

(a) (4 pts.) Using the identities

i. $\nabla \rho = -\frac{1}{c}\dot{\rho}\,\hat{\mathscr{r}}$ (here $\rho = \rho(\mathbf{r}', t_r)$)

ii. $\nabla \cdot (\hat{\mathscr{r}}/\mathscr{r}) = 1/\mathscr{r}^2$

iii. $\nabla (1/\mathscr{r}) = -\hat{\mathscr{r}}/\mathscr{r}^2$

iv. $\nabla \cdot (\hat{\mathscr{r}}/\mathscr{r}^2) = 4\pi\delta^3(\vec{\mathscr{r}})$,

and the retarded form of the potential

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int d\tau' \frac{\rho(\mathbf{r}', t_r)}{\mathscr{r}}, \tag{2}$$

show explicitly that $\Box^2 V = -\frac{1}{\epsilon_0}\rho$.

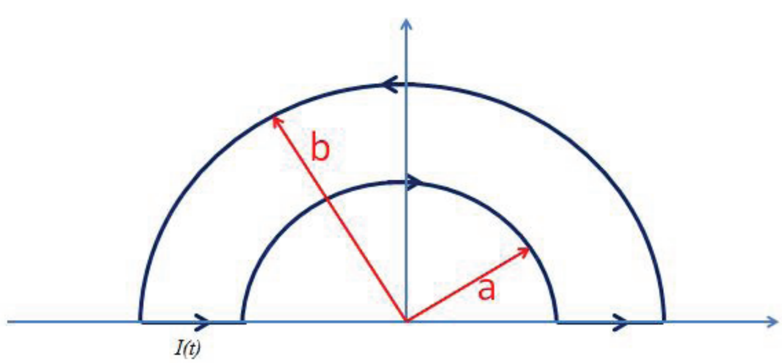

Figure 3: Wire loop with current $I(t)$ as shown.

The next questions are based on the figure, where a wire is bent into a loop as shown in the figure. Label the two arcs and two segments clearly with numbers 1-4. The goal is to calculate the potentials and fields at $\vec{r} = 0$ (center of arcs) for a current increasing as $I = kt$, where k is a constant. There is no net charge on the wire.

(b) (4 pts.) Calculate $\mathbf{A}(\mathbf{r} = 0, t)$ and $V(\mathbf{r} = 0, t)$.

(c) (4 pts.) Calculate $\mathbf{E}(\mathbf{r} = 0, t)$.

(d) (3 pts.) Explain how you would go about calculating $\mathbf{B}(\mathbf{r} = 0, t)$, but do not do it!