

PHY4324–Electromagnetism II

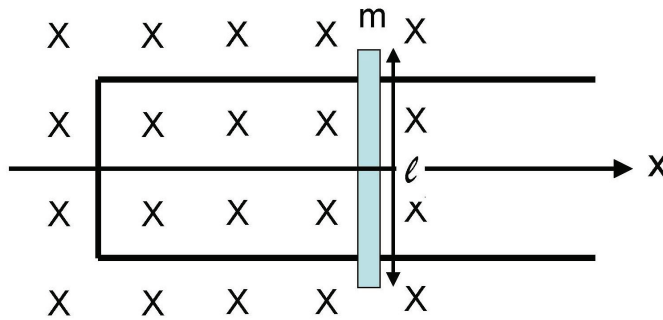
Fall 2012

Test 1 – 55 minutes

Sept. 24, 2012

No other materials except calculators and one handwritten formula sheet allowed. If you can't do one part of a problem, solve subsequent parts in terms of unknown answer—define clearly. Do 3 of the first 4 problems, CLEARLY indicating which you want graded by circling the problem number!. Each problem is worth 10 pts., for maximum of 30 points.

1. Sliding rail. A metal bar (mass m , length ℓ , cross-sectional area A , conductivity σ) slides on a horizontal, frictionless, perfectly conducting rail in a uniform magnetic field, as shown in the figure below.



Assume the bar slides always perpendicularly to the rails.

(a) (2 pts.) If the bar moves with velocity v in the $+x$ direction, what is the current in the circuit (direction as well as magnitude)?

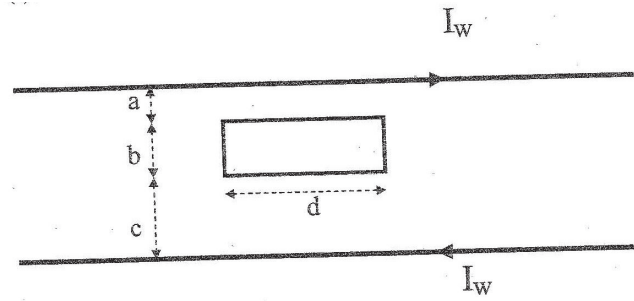
(b) (2 pts.) In the same situation as (a), what is the force on the bar? Be sure to give the direction as well as the magnitude

(c) (2 pts.) If the initial velocity of the bar is v_0 at time $t = 0$, what is the expression for the velocity at later times t ?

(d) (2 pts.) If the initial velocity of the bar is v_0 , how far does the bar slide before stopping?

(e) (2 pts.) Show by explicit integration that the energy produced in Joule heating is equal to the initial kinetic energy of the bar.

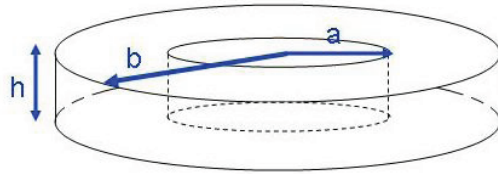
2. Mutual/self-inductance. Consider the circuit shown in the figure below, consisting of two very long wires with a rectangular loop located in between them. A current I_W is flowing in the long wires in the direction indicated.



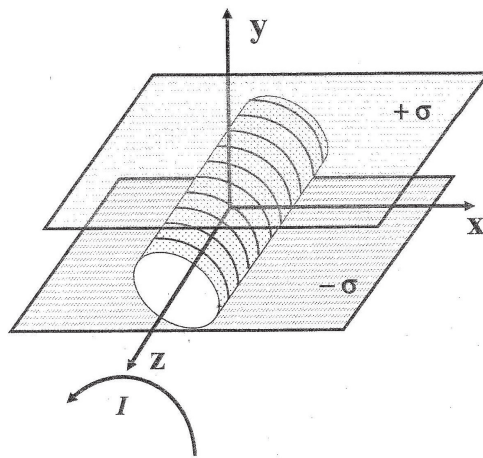
- (a) (2 pts.) Find the flux through the wire loop.

- (b) (4 pts.) Find the mutual inductance of the system (wires-loop)

- (c) (4 pts.) A toroidal coil has a rectangular cross section (see below) with inner radius a , outer radius b , height h with N total turns. Compute the self-inductance of such a coil.



3. Between the sheets. An infinite solenoid with radius R , n turns per unit length and carrying current I (going counterclockwise in figure as shown) is centered on the z axis. It is sandwiched between two infinite sheets of charge separated by a distance d with surface charge density σ for the $y = +d/2$ plane and $-\sigma$ for the $y = -d/2$ plane as shown.



- (a) (2 pts.) Compute \mathbf{E} and \mathbf{B} throughout *all* space.

(b) (4 pts.) Compute the Maxwell stress tensor in *all* regions of space.

(c) (4 pts.) Where in space is momentum stored in the electromagnetic fields?
Give the value of the momentum density $\varphi(x, y, z)$ in that region.

4. *EM wave.*

(a) (4 pts.) (Short answer)

i. Is an EM wave longitudinal or transverse? Justify your answer.

ii. True or false? “A general EM wave must be proportional to $\sin(kx - \omega t)$ or $\cos(kx - \omega t)$.” Justify.

iii. True or false? “An EM wave requires a medium in which to propagate.”
Justify.

iv. Describe what is meant by a circularly polarized wave.

(b) (3 pts.) Use Maxwell's equations to prove that wavelike solutions exist in a region of space with no charges or currents, and that this wave travels at a particular speed given by electromagnetic constants. What is this speed?

(c) (3 pts.) Use Faraday's law to show that in a monochromatic plane electromagnetic wave $\mathbf{E} = \mathbf{E}_0 e^{i(kz - \omega t)}$ and $\mathbf{B} = \mathbf{B}_0 e^{i(kz - \omega t)}$ fields are always mutually perpendicular (\mathbf{E}_0 and \mathbf{B}_0 are constant vectors).