

observer was tested at frequencies of 5, 10, and 15 Hz. Settings fell within 0.01 log unit of baseline with the exception of the 5-Hz dark-adapted condition, where there was a shift of less than 0.04 log unit in the direction consistent with a slight negative blue-cone contribution.

Might a significant blue-cone contribution be present under conditions different from the ones investigated here? We consider it unlikely. The 439-nm standard was roughly 25 times more effective in quanta absorbed/deg²/s for the blue cones than the 492-nm test thus giving an effective blue cone modulation of more than 90%. Use of an even shorter wavelength standard could have provided only slightly greater blue-cone excitation at a given photopic luminance. Use of a long wavelength background, however, could favor the blue cones appreciably more by selectively desensitizing the red and green cones. With this in mind we repeated the basic experiment on AE with a 60 td 563 nm background always present and found still no evidence of any blue-cone contribution.

The results, therefore, show beyond reasonable doubt that the blue cones make no significant contribution to luminance.

ACKNOWLEDGMENT

This work was supported by NIH Grant No. EY 01711. We thank R. M. Boynton for comments.

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⁸These wavelengths were selected for each observer as producing the same ratio of red to green cone excitation on the basis of their similar appearance when viewed side by side in a bipartite field upon a 30 td 420 nm background which desensitized the blue cones. When AE searched for a wavelength to match 439 nm under these conditions, the range of matches included 492 nm. 439 nm and 492 nm also agree well with Walraven's summary of tritanopic metamers (Ref. 2) as well as being concordant with Smith and Pokorny's estimates of *M* and *L*. For the anomalous observer, 510 nm was near the center of the matching range. We determined that a 30 td 420 nm background has at most a small effect on the relative red versus green cone contributions to luminance, so the choice of wavelengths is not very critical: an error of up to 10 nm in the chosen test for the normal observers would produce only negligible deviations in the data. The latitude for the deuteranomalous observer is even greater.

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¹⁰This was ascertained by recording thresholds as a function of time after the bleaching exposure, for a 420-nm test spot upon a 220 td 574 nm background.

¹¹In making these predictions we assumed that flicker is minimized when

$$k_R R_{439} + k_G G_{439} + k_B B_{439} = c(k_R R_\lambda + k_G G_\lambda + k_B B_\lambda)$$

where *k_R*, *k_G*, and *k_B* reflect red, green, or blue cone sensitivity in the presence of some background and *c* reflects the radiance the observer must set to minimize flicker. *k_R* is about equal to *k_G*. *R_λ*, *G_λ*, and *B_λ* are taken from either Vos and Walraven¹ or Walraven.² We estimated the depression in red and green cone sensitivities by determining thresholds for detecting 2-Hz flicker of a 492-nm test upon the 420-nm background. We found a total depression, at the highest background luminance, of about 0.4 log unit, to about 40% of the dark adapted sensitivity. To estimate the depression in sensitivity of the blue cones we noted the effects of the various backgrounds on the visibility of 2-Hz chromatic flicker produced by an alternation of standard and equiluminous test. The intensities required for detecting chromatic changes at 2-Hz varied by a factor of at least 8 for AE and at least 10 for DM as a function of background intensity. Thus the background did fulfill its intended function of selectivity depressing blue cone sensitivity.

Far-infrared ordinary-ray optical constants of quartz

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(Received 20 March 1979; revised 9 July 1979)

By using asymmetric Fourier-transform techniques, the room-temperature optical constants of z-cut quartz have been determined between 100 and 500 cm⁻¹.

The infrared and far-infrared properties of crystal quartz have been studied a number of times over a period of 50 years.^{1–17} There are two reasons for this attention. The lattice dynamics of this material are quite interesting, par-

ticularly at high temperatures. In addition, quartz is one of the most useful window materials in the far-infrared, with a high transparency at room temperature for frequencies smaller than 250 cm⁻¹.

The optical properties have been determined in various ways by previous investigators. In the far infrared, measurements of the power transmission of crystalline samples provide the absorption coefficient (or, equivalently, the extinction coefficient), while analysis of the interference pattern arising from multiple internal reflections in samples with parallel surfaces gives the index of refraction. This procedure, which has been used by Roberts and Coon,⁶ Berman and Zhukov,¹⁰ Plendl *et al.*,¹² and Loewenstein *et al.*¹⁶ at frequencies smaller than about 350 cm^{-1} , has the advantage of being experimentally straightforward and easily analyzed, but does not work in regions of high absorption. At higher frequencies, in the optical phonon region, measurements of the power reflectance can be followed by either a fit to a series of oscillators to find classical dispersion parameters or by a Kramers-Kronig analysis to obtain the phase shift and the optical constants. Reflectance measurements have been carried out for quartz by Spitzer and Kleinman,⁴ Merten,¹⁴ Onstott and Lucovsky,¹⁵ and Gervais and Piriou¹⁷ for frequencies between 300 and 2000 cm^{-1} .

Chamberlain *et al.*⁸ and Russell and Bell¹³ studied the optical properties of crystal quartz using a third technique, namely, asymmetric Fourier-transform spectroscopy wherein the sample is introduced into one arm of a Michelson interferometer. Russell and Bell¹³ obtained the index of refraction and absorption coefficient between 20 and 370 cm^{-1} for both the ordinary and the extraordinary ray in quartz. In this Letter we report a measurement of the ordinary-ray optical constants of z-cut quartz between 100 and 500 cm^{-1} using the asymmetric Fourier-transform technique. A least-squares fit to a sum of Lorentzian oscillators gives the oscillator parameters for the observed optical phonons in the $350\text{--}500\text{ cm}^{-1}$ region, a region that has not previously had much experimental attention.

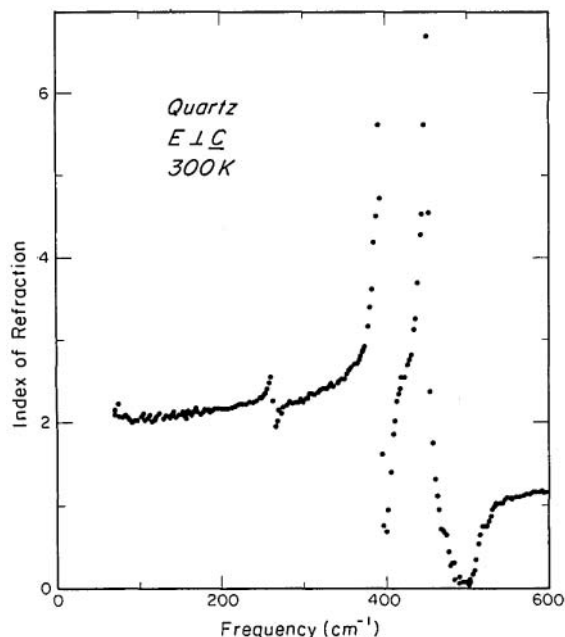


FIG. 1. Index of refraction for z-cut crystal quartz between 100 and 500 cm^{-1} . The data points are shown at a resolution of 3.2 cm^{-1} . The scatter in the smooth portion of the curve is indicative of the signal-noise ratio in the measurement.

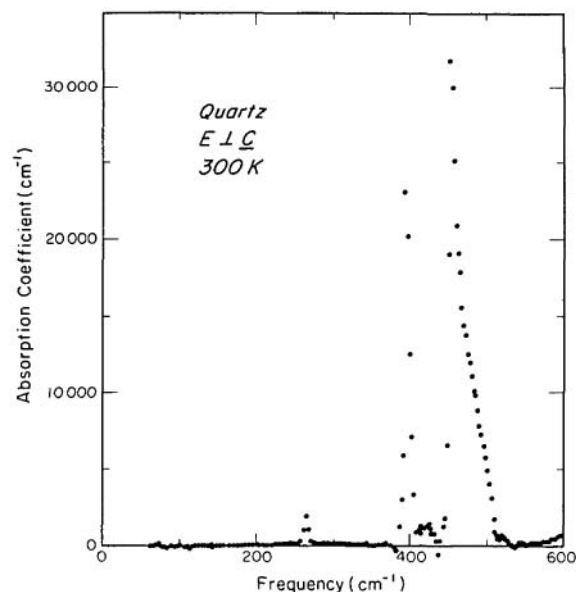


FIG. 2. Absorption coefficient for z-cut crystal quartz between 100 and 500 cm^{-1} . The maximum value obtained for the absorption coefficient is $33\,000\text{ cm}^{-1}$ at 440 cm^{-1} . The negative values observed near the strong lines arise because no apodization was used on the interferogram.

Asymmetric interferometry in the far infrared was developed by Chamberlain *et al.*^{8,18} and by Bell.¹⁹ In this technique,²⁰ the sample under investigation is placed in one arm of a Michelson interferometer where it affects both the amplitude and the phase of the radiation traveling in that arm. The interference fringes produced by the interferometer are changed both in intensity and position by the introduction of the sample.

The interferometer used in these measurements is the one built and described by Russell and Bell.²¹ A beam splitter of $2.5\text{-}\mu\text{m}$ thick Mylar and a Golay detector with a diamond window were used in these measurements. With a black polyethylene transmission filter, this combination provided far-infrared energy between 100 and 500 cm^{-1} .

The single-crystal z-cut quartz plate used in these studies was obtained from the Karl Lambrecht Corporation. In z-cut quartz the crystallographic c axis is normal to the surface of the crystal so that the optical properties are governed by the ordinary-ray complex refractive index. The ordinary-ray index of refraction of this specimen between 75 and 550 cm^{-1} is shown in Fig. 1 and the absorption coefficient $\alpha = 2\omega k/c$ over the same frequency region is shown in Fig. 2. The data are shown at a resolution of 3.2 cm^{-1} without apodization. The index of refraction near 150 cm^{-1} is 2.14, in reasonably good agreement with the data of Russell and Bell.¹³ The signal-to-noise ratio is not good enough to see the weak structure at 128 cm^{-1} . Absorption maxima, with corresponding structure in the refractive index, are seen at 263, 393, and 450 cm^{-1} . The absorption coefficient is greater than 100 cm^{-1} for frequencies greater than 250 cm^{-1} , so this frequency is an effective upper limit for the transmission of all but very thin specimens.

The dielectric response of an insulator is often described by a complex dielectric function which is a sum of Lorentzian oscillators,

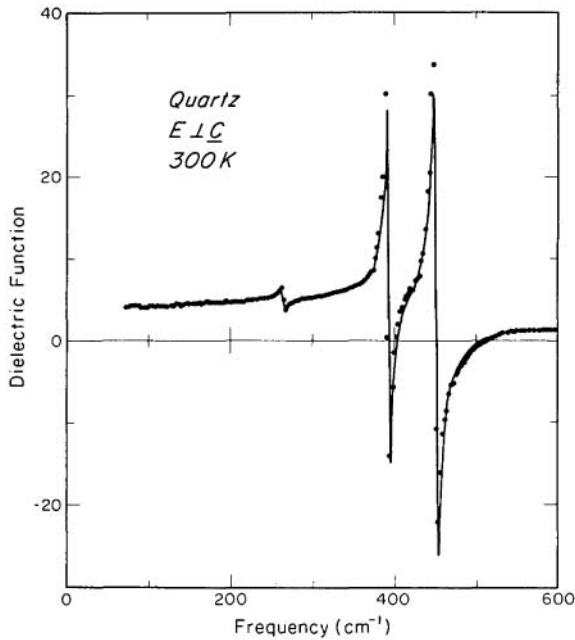


FIG. 3. Real part of the dielectric function for z-cut crystal quartz between 100 and 500 cm^{-1} . The solid line is a fit to a sum of three oscillators, using parameters given in Table I.

$$\epsilon(\omega) = \epsilon_{\infty} + \sum_{j=1}^N \frac{s_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j}, \quad (1)$$

where ϵ_{∞} is the contribution of high-frequency (electronic) transitions, the sum runs over the symmetry-allowed infrared active vibrational modes of the lattice, ω_j is the frequency of the j th transverse-optic mode, s_j is its strength, and γ_j is its full width at half maximum. For the ordinary ray in crystal

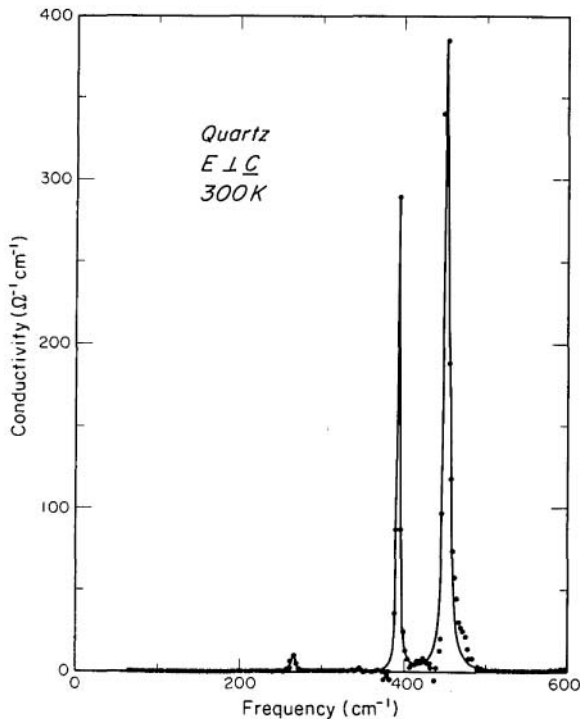


FIG. 4. Frequency-dependent conductivity for z-cut crystal quartz between 100 and 500 cm^{-1} . The solid line is obtained from a sum of three oscillators, using parameters given in Table I.

TABLE I: Oscillator parameters for low frequency ordinary-ray optical phonons in quartz.

Frequency (cm^{-1})	Width ^a (cm^{-1})	Strength	Strength		
			Ref. 4	Ref. 13	Ref. 17
263.4 ± 0.3	5.6	0.032 ± 0.001	—	0.030	—
392.7 ± 0.3	3.2	0.37 ± 0.01	0.33	(0.36) ^b	0.33
450.2 ± 0.3	5.9	0.75 ± 0.05	0.82	(0.85) ^b	0.83

^a Value for width partially determined (especially for 393- cm^{-1} line) by instrumental resolution of 3.2- cm^{-1} .

^b Corrections to values of Ref. 4 suggested by Russell and Bell¹³ so as to obtain measured far infrared value for refractive index.

quartz, where the electric field is perpendicular to the crystallographic c axis, there are eight of these phonons.^{13,14,17,22}

The complex dielectric function is related to the complex refractive index by $\epsilon = N^2$ and can be written as

$$\epsilon = \epsilon_1 + 4\pi i \sigma_1 / \omega, \quad (2)$$

where $\epsilon_1(\omega)$ is the real dielectric function and $\sigma_1(\omega)$ is the frequency-dependent conductivity. Figures 3 and 4 show $\epsilon_1(\omega)$ and $\sigma_1(\omega)$, respectively. The data are shown as points, and fits to Eq. (1) as the solid lines. These fits are done as a nonlinear least-squares procedure²³ directly to $\epsilon_1(\omega)$ or to $\sigma_1(\omega)$. Because the wings of the 393- and 450- cm^{-1} lines overlap, we have made a six-parameter fit to this doublet, whereas the 263- cm^{-1} line was fitted separately.

The parameters obtained by this fitting procedure are given in Table I. We list the frequency ω_j , the width γ_j , and the strength s_j of each mode in the first three columns. The next three columns of this table give the oscillator strengths obtained by three previous investigators.^{4,13,17} Our results show no significant difference in the frequencies or widths of these modes. On the other hand, there are differences in the oscillator strengths. Russell and Bell¹³ have suggested that the strengths obtained by Spitzer and Kleinman⁴ for the 393 and 450- cm^{-1} modes should be increased to give agreement between the static dielectric constant calculated from Eq. (1) and their value determined from extrapolation of the far-infrared refractive index to zero frequency. Our results for the 393- cm^{-1} line are in accord with this suggestion. For the 450- cm^{-1} line our results are smaller than previously found.^{4,17} However, this value has a larger uncertainty because of a low signal for frequencies greater than 430 cm^{-1} in the present experiment.

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Increased spectroscopic resolution with separated gratings and prisms

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(Received 13 July 1979)

The use of two separated diffraction gratings or prisms doubles the spectroscopic resolution compared to that obtained with a single equivalent grating or prism. Ultra-high resolution is possible with widely spaced gratings. Varying the spacing between the two gratings or prisms produces useful modulation in the spectrum.

INTRODUCTION

It is well known that the use of separated receiving elements in a Michelson stellar interferometer (in either the optical- or radio-frequency domains) yields high angular resolution without the need for a single large receiving element. However, the analogous techniques for high spectral resolution using separated diffraction gratings or prisms do not seem to have been described in the literature.¹ In this Letter I point out that the use of separated diffraction gratings and separated segments of prisms leads to high spectral resolution without the need for a single large grating or prism. I also show that varying the spacing between the gratings or prisms produces useful modulation in the spectrum.

I. SEPARATED GRATINGS

The intensity pattern expected from two separated gratings may be calculated using standard scalar diffraction theory.^{2,3} Let W be the distance between the centers of the two gratings. Each grating has M rulings spaced a distance d apart. (See Fig. 1.)

For collimated light incident at an angle θ_0 with respect to the normal to the plane containing the two gratings, the intensity pattern $I_2(\theta)$ of the diffracted light of wavelength λ is given by the expression

$$I_2(\theta) = a^2 [\sin^2 M\gamma / \sin^2 \gamma] 4 \cos^2 N\gamma, \quad (1)$$

where $N = W/d$ is the number of rulings in a grating of width W with the same ruling spacing d . γ is defined by

$$\gamma = \pi d (\sin \theta_0 - \sin \theta) / \lambda,$$

and a^2 is the "form factor" for diffraction from a single grating ruling. The term in brackets is the diffraction pattern from a single grating with M rulings.

The corresponding expression for the intensity pattern due to a single grating of width W is

$$I_1(\theta) = a^2 \sin^2 N\gamma / \sin^2 \gamma. \quad (2)$$

In both cases, maxima in the diffraction pattern occur when $\gamma = m\pi$, with $m = 0, \pm 1, \pm 2, \dots$. The integer m labels the order of interference.