Investigation of Self-Induced Depolarization of Laser Radiation in Terbium Gallium Garnet

Efim A. Khazanov, Oleg V. Kulagin, Sanichiro Yoshida, David B. Tanner, and David H. Reitze

***Abstract*— Absorption of laser radiation in magnetooptical materials results in a temperature gradient which induces depolarization due to both the temperature dependence of the Verdet constant and the photoelastic effect of thermal strains. This results in a limitation of the isolation ratio of Faraday isolators in high average power lasers. Here, we derive expressions for the isolation ratio as a function of beam power, beam radius, angle between incident polarization and crystal axis, and characteristics of the magnetooptical material. The theoretical results are compared with experiments for a terbium gallium garnet crystal. Our results allow us to choose the optimal parameters to maximize the isolation ratio and to compare different materials from this point of view.**

***Index Terms*—Depolarization, Faraday effect, photoelastic effect, Verdet constant.**

I. Introduction

FARADAY isolators are a key optical element for many laser-based applications. In particular, they are often used in laser applications that utilize high average power in which large isolation ratios and high-quality optical wavefronts must be maintained. Such requirements exist, for example, in repetitively pulsed lasers with phase-conjugate mirrors [1] and in large-scale interferometric gravitational wave detectors (such as those being developed by LIGO, VIRGO, GEO, TAMA, and ACIGA) which will use frequency- and amplitude-stabilized high-power lasers for sensitive readout of the gravitational wave signal [2]–[7]. Terbium gallium garnet (TGG) is often used as an optical element of Faraday isolators in such lasers because of its high thermal conductivity and large Verdet constant. Previous investigations on high-power propagation effects of TGG have been limited to thermal lensing [8], which does not directly affect the isolation ratio. By virtue of its high thermal conductivity, TGG is usually preferred over Faraday glass for high-power applications. Here, we report on theoretical and experimental investigations of self-induced depolarization of laser radiation in TGG.

Physically, self-induced depolarization is caused by absorption of laser radiation. This absorption results in a spatially nonuniform temperature distribution that compromise the isolation ratio in two ways. The first effect is a change in the angle of linear polarization rotation due to the temperature dependence of the Verdet constant. This introduces a nonzero electric field vector in the orthogonal polarization channel, resulting in a reduction in both the optical throughput and the isolation ratio with respect to the initial alignment. The second effect is birefringence due to the photoelastic effect of thermal strains [9]. In this case, initially isotropic media experience linear birefringence and initially circular eigenpolarizations (Faraday effect) become elliptical ones. The angle of incline of their axes, their ellipticity, and phase delay between them vary over the cross-sectional area of the rod such that, at each point of the cross section, the beam polarization changes. As a result, a polarized input beam becomes depolarized at the output. Birefringence due to the photoelastic effect of thermal strains has been investigated for cases where the optical element is isotropic or has linear birefringence in the absence of temperature gradient [9], [10]. To our knowledge, the self-induced depolarization and consequent degradation of the isolation ratio has never been investigated.

In this paper, we investigate the effects of self-induced depolarization due to thermal strain and temperature-dependent changes in the Verdet constant. In Section II, we derive expressions for the isolation ratio as a function of beam power, beam radius, angle between the incident polarization and the crystal axis, and the characteristics of the magnetooptical material. Influences of the above-mentioned two effects are analyzed. The influence of longitudinal magnetic field variations on depolarization is investigated theoretically. In Section III, the theoretical results are compared with experiments both with and without a magnetic field. In Section IV, we discuss our results and their consequences for isolation of CW and repetitively pulsed high-power lasers. In particular, we find that the depolarization is predominantly due to thermally induced linear birefringence. Finally, we conclude in Section V.

II. Theoretical Approach

To compute the isolation ratio, we assume that the laser beam has a Gaussian spatial profile and propagates through a cylindrically symmetric geometry. We consider the general case in which the input polarization, crystalline axes, and eigenpolarizations are distinct. We first formulate the Jones matrix at a given point of the cross section of the crystal. Then, using this Jones matrix and a linearly polarized incident beam, we evaluate what portion of the output intensity is depolarized. Integration of this depolarized intensity over the cross section gives us the depolarization ratio γ. The isolation ratio is evaluated as $1/\gamma$. From this expression, we derive how the magnitude of the depolarization depends on

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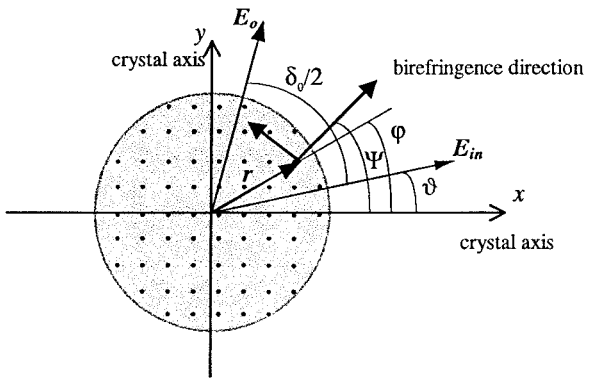

Fig. 1. Crystal cross section. Crystal axes are along the x and y axes. θ is the angle between incident polarization $\boldsymbol{E}_{\rm in}$ and the crystal axis. Ψ is the angle between the eigen, linear polarization and the crystal axis. φ is the polar angle. $\delta_0/2$ is the angle between the incident and the reference polarizations. Due to the Faraday effect, the polarization rotates counterclockwise.

effects which result from the absorption of laser radiation and material properties of the crystals. Finally, we consider how the longitudinal spatial dependence of the magnetic field affects the depolarization and isolation ratio.

A. General Theory for Computing Depolarization

We let the crystal axes coincide with the x and y axes, and the polarization of the incident beam be tilted with respect to the x axis by an angle θ (see Fig. 1)

$$\boldsymbol{E}_{\rm in} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}. \tag{1}$$

The total birefringence at radius $\boldsymbol{r}$ is the superposition of the circular birefringence (the Faraday effect) and the linear birefringence. The former is characterized by the phase delay between the two circular eigenpolarizations, $\delta_c(\boldsymbol{r})$, while the latter is characterized by the phase delay between the two linear eigenpolarizations $\delta_l(\boldsymbol{r})$ and by the angle between one of the linear eigenpolarization and the x axis, $\Psi(\boldsymbol{r})$.

The Jones matrix $F(\boldsymbol{r})$ can be found as (hereafter, we omit the argument $\boldsymbol{r}$)

$$F = M(-\Psi) F_{xy} M(\Psi) \tag{2}$$

where M is the matrix representing a rotation of angle ψ

$$M = \begin{pmatrix} \cos\Psi & \sin\Psi \\ -\sin\Psi & \cos\Psi \end{pmatrix} \tag{3}$$

and F_{xy} is the Jones matrix for an optical element having circular birefringence and linear birefringence along the x and y axes [11]

$$F_{xy} = \begin{pmatrix} \cos\dfrac{\delta}{2} - i\dfrac{\delta_l}{\delta}\sin\dfrac{\delta}{2} & -\dfrac{\delta_c}{\delta}\sin\dfrac{\delta}{2} \\ \dfrac{\delta_c}{\delta}\sin\dfrac{\delta}{2} & \cos\dfrac{\delta}{2} + i\dfrac{\delta_l}{\delta}\sin\dfrac{\delta}{2} \end{pmatrix}. \tag{4}$$

Here δ is the phase shift between elliptical eigenpolarizations, which is the quadrature sum of the linear and circular phase shifts

$$\delta^2 = \delta_l^2 + \delta_c^2. \tag{5}$$

Equation (5) is valid when the phase retardation caused by the combination of the linear and circular birefringence is much smaller than the absolute phase of the light [11]. Physically, this corresponds to an overall phase shift of many cycles as the light propagates through the crystal.

From (2)–(4), the following equation for F can be obtained:

$$F = \sin\frac{\delta}{2} \cdot \begin{pmatrix} \operatorname{ctg}\dfrac{\delta}{2} - i\dfrac{\delta_l}{\delta}\cos 2\Psi & -\dfrac{\delta_c}{\delta} - i\dfrac{\delta_l}{\delta}\sin 2\Psi \\ \dfrac{\delta_c}{\delta} - i\dfrac{\delta_l}{\delta}\sin 2\Psi & \operatorname{ctg}\dfrac{\delta}{2} + i\dfrac{\delta_l}{\delta}\cos 2\Psi \end{pmatrix}. \tag{6}$$

To evaluate depolarization, it is necessary to compare the elliptical output polarization

$$\boldsymbol{E}_{\rm out} = F\boldsymbol{E}_{\rm in} \tag{7}$$

with a reference linear polarization. This reference polarization should represent the output under the ideal situation where the TGG crystal does not have linear birefringence that causes the ellipticity ($\delta_l = 0$). However, since the Verdet constant depends on the temperature of the crystal which is a function of $\boldsymbol{r}$, the angle of rotation $\delta_c/2$ in this "ideal" situation depends on $\boldsymbol{r}$, making it impossible to determine such a reference polarization uniquely over the whole cross-sectional area. Thus, we define a reference polarization $\boldsymbol{E}_o$ using a variable angle of rotation $\delta_o/2$ (see Fig. 1) that can be adjusted in such a way that the resultant depolarization may be minimized (see below). By this definition, the reference polarization $\boldsymbol{E}_o$ can be written as

$$\boldsymbol{E}_o = \begin{pmatrix} \cos\left(\dfrac{\delta_0}{2} + \theta\right) \\ \sin\left(\dfrac{\delta_0}{2} + \theta\right) \end{pmatrix}. \tag{8}$$

In the limit that the Verdet constant and hence δ_c approaches zero, $\delta_o/2$ is 0 and $\boldsymbol{E}_o$ becomes $\boldsymbol{E}_{\rm in}$. The fraction of energy in $\boldsymbol{E}_{\rm out}$ that remains in the polarization $\boldsymbol{E}_o$ can be calculated as

$$\chi = \frac{|\boldsymbol{E}_{\rm out}\boldsymbol{E}_0^*|^2}{|\boldsymbol{E}_{\rm out}|^2 \cdot |\boldsymbol{E}_0|^2}. \tag{9}$$

Equations (1) and (6)–(9) lead to

$$\begin{aligned} 1 - \chi = {} & \sin^2\left(\frac{\delta - \delta_0}{2}\right) - \sin^2\left(\frac{\delta_0}{2}\right)\sin^2\left(\frac{\delta}{2}\right) \cdot \left(1 - \frac{\delta_c}{\delta}\right)^2 \\ & + 2\cos\frac{\delta - \delta_0}{2}\sin\left(\frac{\delta_0}{2}\right)\sin\left(\frac{\delta}{2}\right)\left(1 - \frac{\delta_c}{\delta}\right) \\ & - \frac{\delta_l^2}{\delta^2}\sin^2\left(\frac{\delta}{2}\right)\cos^2\left(2\Psi - 2\theta - \frac{\delta_0}{2}\right). \end{aligned} \tag{10}$$

Note that in the absence of the circular birefringence ($\delta_c = 0, \delta_0 = 0$) (10) reduces to the formula representing the case where linear birefringence alone is present [10], [12]

$$1 - \chi_0 = \sin^2\left(\frac{\delta_l}{2}\right)\sin^2(2\Psi - 2\theta). \tag{11}$$

The depolarization ratio γ is defined as the power in the undesired polarization normalized by the total beam power.

Thus

$$\gamma = \frac{\int (1-\chi)\exp\left(-\frac{r^2}{r_0^2}\right) d\boldsymbol{r}}{\pi r_0^2} \tag{12}$$

where we assume that the laser beam is Gaussian

$$I(r) = I_0 \exp\left(-\frac{r^2}{r_0^2}\right)$$

where $r = |\boldsymbol{r}|$. The isolation ratio is calculated as $1/\gamma$.

For further analysis, let us assume that the linear birefringence is much smaller than the circular birefringence (the estimations below justify this assumption for very high power levels)

$$\frac{\delta_l}{\delta_c} \ll 1 \tag{13}$$

and the angle of rotation $\delta_c/2$ is close to the angle $\delta_0/2$ corresponding to the above-defined reference polarization $\boldsymbol{E}_o$

$$\delta_c - \delta_0 \ll 1. \tag{14}$$

Taking into account (13) and (14) in (10) and (12), we obtain the formulas representing the depolarization ratio within an accuracy on the order of $(\delta_l/\delta_c)^4$ and $(\delta_l/\delta_c)^2(\delta_c - \delta_0)$

$$\gamma = \gamma_V + \gamma_P \tag{15}$$

$$\gamma_V = \frac{1}{4\pi \cdot r_0^2} \int (\delta_c - \delta_0)^2 \exp\left(-\frac{r^2}{r_0^2}\right) d\boldsymbol{r} \tag{16}$$

$$\gamma_P = \frac{\sin^2(\delta_0/2)}{\delta_0^2 \cdot \pi r_0^2} \int \delta_l^2 \sin^2\left(2\Psi - 2\theta - \frac{\delta_0}{2}\right) \exp\left(-\frac{r^2}{r_0^2}\right) d\boldsymbol{r}. \tag{17}$$

As we can see from (15), the depolarization ratio is the sum of two terms γ_V and γ_P. The first term represents the part of the depolarization due to the temperature dependence of angle $\delta_c/2$ caused by the temperature-dependent Verdet constant. It is totally independent of the linear birefringence (δ_l, Ψ). Note that, if δ_c does not depend on $\boldsymbol{r}$, it is always possible to make $\gamma_v = 0$ by choosing δ_0 appropriately, and that δ_v does not depend on angle θ. The second term in (15) represents the change of eigenpolarizations caused by the photoelastic effect, which can either be initially present (due to crystal strain) or be self-induced by the laser beam via the induced temperature gradient. This term is a periodic function of θ with a period $\pi/2$; hence, γ is a periodic function of θ with the same period.

B. Dependence of Depolarization on Laser Absorption

To consider self-induced effects, we must solve a thermodiffusion equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\alpha \cdot I_0}{\kappa} \exp\left(-\frac{r^2}{r_0^2}\right) \tag{18}$$

where α and κ are the absorption coefficient and thermal conductivity of the medium, respectively. Taking into account that

$$\frac{dT}{dr}(r=0) = 0$$

we obtain the following equation from (18):

$$\frac{dT}{dr} = -\frac{\alpha P_0}{2\pi\kappa} \cdot \frac{1-\exp(-r^2/r_0^2)}{r} \tag{19}$$

where $P_0 = \pi r_0^2 I_0$ is the total beam power. The temperature gradient in the crystal with [001]-orientation induces linear birefringence with phase delay δ_l and the angle of the eigen axis Ψ, which vary over polar coordinates $\boldsymbol{r}$ and φ [10]:

$$\delta_l(r,\varphi) = 4\pi\frac{L}{\lambda}Q\left[\frac{1+\xi tg^2(2\Psi)}{1+tg^2(2\Psi)}\right]^{1/2} \cdot \frac{1}{r^2}\int_0^r r^2\frac{dT}{dr}dr$$
$$\tan[2\Psi(\varphi)] = \tan(2\varphi)\sqrt{\xi} \tag{20}$$

where

$$\xi = \left(\frac{2p_{44}}{p_{11}-p_{12}}\right)^2, \quad Q = \left(\frac{1}{L}\frac{dL}{dT}\right)\frac{n_0^3}{4}\frac{1+\nu}{1-\nu}\cdot(p_{11}-p_{12})$$

and L is a crystal length, ν is the Poisson coefficient, and $p_{i,j}$ are photoelastic coefficients. Here we took into account that the crystallographic axes coincide with the x and y axes. From (17), (19), and (20), we obtain

$$\gamma_P = \left[\frac{L\alpha P_0 Q}{\lambda\kappa}\right]^2 \frac{A_1}{8}\frac{\sin^2(\delta_0/2)}{(\delta_0/2)^2}\left(1+(\xi-1)\cos^2\left(2\theta+\frac{\delta_0}{2}\right)\right) \tag{21}$$

where

$$A_1 = \int_0^\infty (1/y - \exp(-y)/y - 1)^2 \exp(-y)\, dy \cong 0.137.$$

Note that, if the circular birefringence is absent (i.e., $\delta_c = 0$), the depolarization ratio γ_0 from (11), (12), (19), and (20) becomes

$$\gamma_0 = \left[\frac{L\alpha P_0 Q}{\lambda\kappa}\right]^2 \frac{A_1}{8}\left(1+(\xi-1)\cos^2(2\theta)\right)$$
$$= \frac{(\delta_0/2)^2}{\sin^2(\delta_0/2)}\gamma_P(\delta_0 = 0). \tag{22}$$

Here we assumed that $\delta_l \ll 1$, as opposed to (13). From (22), we see that the basic features of the depolarization ratio (dependencies on power, beam diameter, wavelength, thermoconductivity, etc.) do not depend on the presence of a magnetic field. If there is no magnetic field, the depolarization is minimal at the optimal value $\theta_{\text{opt}0} = \pi/4 + N\pi/2$, which can be explained as follows. While the temperature gradient and stress are independent of polar angle, there is the preferred direction of the axes of the eigenpolarizations which correspond to $\pi/4 + N\pi/2$ [10] due to the natural orientation of the crystal in these directions (diagonal between crystallographic axes). Hence, the depolarization is minimal when the beam polarization is parallel to one of these directions. With a magnetic field present, there are the preferred (paramount) directions of the axes of the ellipse of the eigenpolarizations for the same reason. In this case, however, the depolarization is minimal at $\theta = \theta_{\text{opt}} = \pi/4 - \delta_0/4 + N\pi/2$, and the beam polarization rotates in the crystal within the range of $\pm\delta_0/4$ around $\theta_{opt0} = \pi/4 + N\pi/2$, the preferred direction of the axes of the ellipse of the eigenpolarizations. Thus, in

comparison with the case when no magnetic field is present, the θ dependence of the depolarization ratio with magnet field is shifted by the angle $\delta_0/4$, which is half of the angle of the polarization rotation.

By rotating the direction of the incident polarization with respect to the crystal axis, which changes θ, we can minimize γ_P (at $\theta = \theta_{opt} = \pi/8 + N\pi/2$)

$$\gamma_P^{\min} = \left[\frac{L\alpha P_0 Q}{\lambda\kappa}\right]^2 \cdot \frac{A_1}{\pi^2}. \tag{23}$$

Here we consider that $\delta_c \cong \pi/2$, assuming that the Faraday rotator rotates the polarization by 45°. Note that, for glass media, $\xi = 1$. Substituting it into (21) we find that, for glass, γ_p does not depend on θ and equals $\gamma_P^{\min}$.

In order to derive γ_V, let us assume that the magnetic field B does not vary over the cross section, and δ_c depends on only r through the temperature dependence of the Verdet constant

$$\delta_c(r) = \text{const} + 2LB\frac{dV}{dT}(T(r) - T(0)).$$

In this case, by varying δ_0, we can minimize γ_V.

Differentiating (16) and equating the derivative to zero, we can find the minimal value of γ_V

$$\gamma_V^{\min} = \left[\frac{\alpha P_0}{16\cdot\kappa}\cdot\frac{1}{V}\cdot\frac{dV}{dT}\right]^2 \cdot A_2 \tag{24}$$

where

$$A_2 = \int_0^\infty f^2(y)\exp(-y)\,dy - \left[\int_0^\infty f(y)\exp(-y)\,dy\right]^2 \approx 0.268$$

$$f(y) = \int_0^y \frac{1-\exp(-z)}{z}\,dz.$$

Here we took into account that $\delta_c \cong \pi/2$. Note that neither γ_P nor γ_V depends on the beam radius r_0 and both are proportional to the square of the incident power P_0. Equations (23) and (24) allow us to compare the limitation of the isolation ratio of Faraday rotators induced by the photoelastic effect and temperature dependence of the Verdet constant

$$\frac{\gamma_V^{\min}}{\gamma_P^{\min}} = \frac{A_2}{A_1}\left[\frac{\pi}{16}\cdot\frac{\frac{1}{V}\frac{dV}{dT}}{Q}\cdot\frac{\lambda}{L}\right]^2 \approx 2\cdot\left[\frac{\pi}{16}\cdot\frac{\frac{1}{V}\frac{dV}{dT}}{Q}\cdot\frac{\lambda}{L}\right]^2. \tag{25}$$

C. Depolarization Dependence on Longitudinal Magnetic Field Variation

We can also take into account the dependence of the magnetic field, hence the circular birefringence, on the longitudinal coordinate z

$$\delta_c(z) = 2V\int_0^z B(z)\,dz, \qquad \delta_c(L) = \delta_c.$$

It is obvious that γ_V does not depend on $B(z)$; however, γ_P does depend on $B(z)$. Using the technique described in [12], we obtain

$$\gamma_P = \frac{\sin^2(\delta_c/2)}{\delta_c^2\cdot\pi r_0^2}\int \delta_l^2 A\sin^2(2\Psi - 2\theta - \beta)\exp\left(-\frac{r^2}{r_0^2}\right)d\mathbf{r} \tag{26}$$

where

$$A = \frac{\delta_c^2}{4\sin^2(\delta_c/2)}\cdot\left(\left[\frac{1}{L}\int_0^L\sin(\delta_c)dz\right]^2 + \left[\frac{1}{L}\int_0^L\cos(\delta_c)dz\right]^2\right)$$

$$\beta = \operatorname{arctg}\left(\frac{\int_0^L \sin(\delta_c)\,dz}{\int_0^L \cos(\delta_c)\,dz}\right).$$

If B is a constant, $A = 1$, and $\beta = \delta_c/2$, then (26) reduces to (17). Equation (26) makes a further analysis very complicated because A and β can vary over the cross section. If they are constant, the dependence of γ on the power, beam diameter, and θ does not change dramatically [compare (17) and (26)]. The influence of the choice of function $\delta_c(z)$ on γ is (including minimization of γ) a subject for future investigation, and here we use (17).

III. Experimental Results

In order to compare our theoretical predictions with experimental data, we measured depolarization in a [001] oriented TGG rod 2 cm in length and 8 mm in diameter as a function of beam diameter, laser power, and polarization angle both with and without a magnet field. The laser used for this measurement was a 9-W CW Nd : YAG laser (Lightwave Electronics, model 220-1064-10 000) having a well-characterized Gaussian intensity profile. We placed the AR-coated TGG rod between a pair of cross-aligned polarizers, where the first polarizer was aligned with the incident polarization. The second polarizer was orthogonal to first one in the case of no magnet field and was adjusted to minimal depolarization in the case where the magnet field was present (see below). The extinction ratio of these polarizers was on the order of 10^{-6} in the absence of the TGG. We measured the power transmitted through the second polarizer (P_t) by a calibrated photodetector and the power incident to the TGG rod P_0 by a power meter. Transmittance of the second polarizer for depolarized beam was close to unity, and we evaluated the depolarization γ as P_t/P_0.

A. Self-Induced Depolarization Measurement without a Magnetic Field

Fig. 2 (filled square data) shows the dependence of the measured γ on the incident laser power P_0 when the bare TGG rod was oriented at the angle for minimum depolarization (corresponding to $\theta = \theta_{\text{opt}0} = \pi/4 + N\pi/2$ in Fig. 1, referred to as the azimuth of minimum depolarization). We found the azimuth of minimum depolarization by a pure rotation of the TGG rod about the beam path until the measured γ was minimized. The background depolarization (P_t/P_0 measured

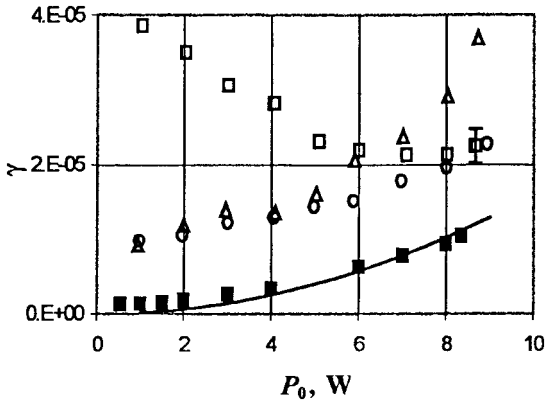

Fig. 2. Dependence γ on P_0 for bare TGG (filled data) and for TGG in a magnetic field (open data) at different adjustments of the second polarizer: at minimum power (triangles), at maximum power (squares), and at each power (circles). The solid line corresponds to the theory for bare TGG at $Q\alpha = 1.2 \times 10^{-8}$/K·cm.

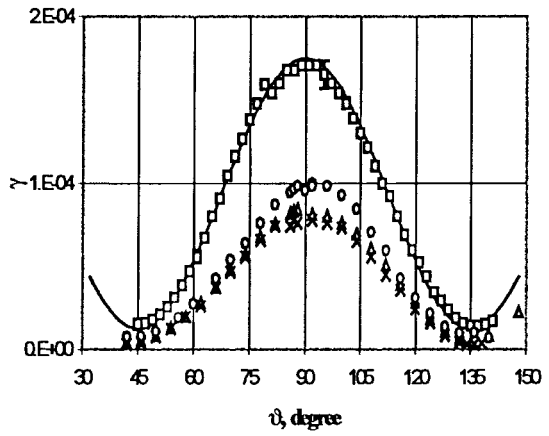

Fig. 3. Dependence of γ on θ for incident powers $P_0 = 8$ W (squares), 5 W (circles), 3 W (triangles), and 0.5 W (crosses) without a magnetic field. The theoretical dependence (solid line) is in arbitrary units at $\xi = 13$.

without placing the TGG rod between the polarizers) was 1.1×10^{-6}. To compare with this measurement, we calculated γ using (22) with $L = 2$ cm, $\lambda = 1.06\,\mu$m, and $\kappa = 7.4$ W/Km. Since Q and α were unknown, we varied $Q\alpha$ as a fitting parameter and fit the resultant curve to the measured γ. A value of $Q\alpha = 1.2 \times 10^{-8}$/K·cm gives the best agreement with theory (Fig. 2). This is somewhat higher than we would expect if we assume that TGG has a Q similar to laser crystals such as Nd : YAG (7×10^{-7}/K) [10] and a textbook absorption of 0.002/cm. A more precise measurement of Q and α requires a more powerful laser or another measuring technique and is a task of future investigation.

We measured γ as a function of angle θ between the beam polarization and the crystal axis. Fig. 3 shows the result obtained at various incident power levels. As predicted by (21), the measured γ shows a period of $\pi/4$. For the data at an incident power of 8 W, we compare the measurement with the theory. Since ξ is unknown, we use it as a fitting parameter. When $\xi \approx 13$, the calculation best fits the measurement (Fig. 3). This value is comparable to that of YAG, $\xi = 10.4$ [9], [10].

We also measured γ at different incident power P_0 and angle θ as a function of the beam radius. The beam radius at the $1/e$ intensity level was varied in the range of 0.5–1.1 mm. Supporting the theoretical considerations discussed above [see (22)], γ is seen to be independent of the beam radius within the accuracy of the experiment.

B. Self-Induced Depolarization Measurement with a Magnetic Field

When the same TGG crystal was placed in the magnet field, the depolarization was dramatically increased at minimum power. This is most likely due to the nonuniformity of the magnetic field in the core and the clamping effect by the housing wall. Nevertheless, γ increased with increasing power, which means that we observed thermal effects. Thus, although when TGG was placed in the magnet field, we cannot quantitatively compare the experimental results with the theory predictions, some quality effects may be investigated.

At high power, we observed that optimization of the angle of the second polarizer may be very efficient from the viewpoint of γ minimization. Comparison of (16) and (17), taking into account (14), shows that depolarization due to the photoelastic effect depends on δ_0 and, hence, the angle of the second polarizer is much weaker than the depolarization due to temperature dependence of the Verdet constant. Therefore, the optimization of γ by adjusting the angle of the second polarizer should be explained from the viewpoint of dependencies of the Verdet constant and the Faraday rotation angle on temperature. The Faraday rotator was designed to rotate the incident polarization by precisely 45° at room temperature. However, due to absorption of laser radiation, the average crystal temperature increases and the average angle of rotation changes due to the temperature dependence of the Verdet constant. This effect corresponds to the first term in (15). It is possible to minimize the depolarization by optimizing the orientation of the second polarizer (optimization of δ_0). The optimal angle depends on the beam power. The minimum γ_V is not equal to zero because the temperature and, hence, the Verdet constant vary by cross sections and can be calculated using (24). Therefore, in the power dependence measurements, if the orientation of the second polarizer was optimized at one incident power, it did not minimize γ at other powers. The three open data plots shown in Fig. 2 indicate this situation.

The open square data points in Fig. 2 represent data taken when the orientation of the second polarizer was adjusted to minimize γ at the maximum incident power, followed by a power reduction without readjustment of the polarizer's orientation. The open triangles depict the case where the direction of the polarizer was adjusted at the lowest power. The open circle data points were obtained when the orientation of the second polarizer was adjusted to minimize the depolarization at each power level of measurement. The difference between the two last cases shows the difference between γ_V when δ_0 precisely equals 45° [see (16)] and the minimum value of γ_V [see (24)]. There is no difference at $P_0 = 1$ W because the average temperature of the crystal is not significantly changed yet, but a greater P_0 results in a greater difference. Fig. 2 is

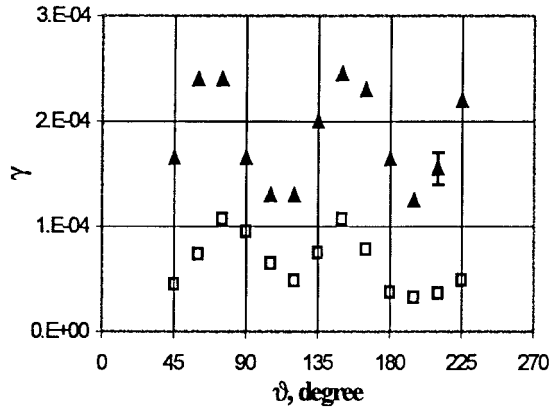

Fig. 4. Dependence of γ on θ for TGG in a magnetic field at the different transverse locations of the laser beam.

significant in that it shows the importance of adjusting the Faraday isolator at each specific power.

All upper three plots shown in Fig. 2 represent the total depolarization: both of the terms in (15) Using these data, we cannot compare terms in (15) to each other and cannot define experimentally which effect is more important for depolarization: the photoelastic effect or the temperature dependence of the Verdet constant. In other words, we cannot compare $\gamma_P^{\min}$ and $\gamma_V^{\min}$ because we had measured only their sum (the open circle data in Fig. 2).

We also measured γ as a function of the angle between the initial polarization and the crystal axis θ when the TGG crystal was placed in the magnet field. This dependence was very sensitive to the alignment of the polarizer and the transverse location of the laser beam. This is probably because there are multiple factors affecting the isolation ratio, such as the nonuniformity of the magnetic field in the core, variance in thermal conductivity, and the clamping effect by the housing wall. Fig. 4 shows examples of these data. However, in most cases, we observed $\pi/4$ period-like θ dependence of γ, and $\theta_{\rm opt}$ is close to $\pi/8 + N\pi/2$. It corresponds to (15)–(17) and (21).

IV. Discussion

Under the assumption that the self-induced linear birefringence in a Faraday isolator is much less than the circular birefringence, the following observations can be made. First, the depolarization ratio γ is proportional to the square of the incident power P_0, but does not depend on the beam radius. This is consistent with the self-similar nature of thermal effects in crystals. Second, the depolarization ratio γ is the sum of two terms γ_V and γ_P which represent the parts of depolarization due to the temperature dependence of the Verdet constant and the photoelastic effect, respectively. For a $[001]$ oriented crystal, γ_P is given by (21) and is a function of the angle between the incident polarization and the crystal axis θ. The peak-to-valley ratio is equal to ξ which is approximately equal to 13 for TGG (see Fig. 3). The minimal value of γ_P is achieved at $\theta = \pi/8 + N\pi/2$ and can be determined by (23). In particular, for glass magnetooptical media, γ_P does not depend on θ given by (23). Third, by adjusting the polarizer's axis, it is possible to achieve the minimum value of γ_V given by (24).

Using these results, we can estimate the relative contributions of γ_V and γ_P. The value of Q is not known for TGG but, for the sake of an estimate, let us use a value for YAG: $Q = 7 \times 10^{-7}$/K. If we use $(1/V)(dV/dT) = 3.5 \times 10^{-3}$/K [13] and $L/\lambda = 20000$ we obtain

$$\frac{\gamma_V^{\min}}{\gamma_P^{\min}} \cong 0.01. \tag{27}$$

If we used the value Q not for YAG but our estimations for TGG (see Section III), the ratio (27) will be even less. Thus, the isolation ratio of high-power Faraday isolators is limited not by the temperature dependence of the Verdet constant, but rather by the depolarization induced by the photoelastic effect. Estimations for glass magnetooptical media such as FR5 Faraday glass using data from [14] and [15] show that the last conclusion is valid not only for TGG but for glass as well.

Taking into account that L is inversely proportional to V and using (23), we can define a figure of merit μ for magnetooptical media—the parameter characterizing it from the viewpoint of the isolation ratio at high average power

$$\mu = \frac{V\kappa}{Q\alpha}. \tag{28}$$

The larger μ is, the better the isolation ratio $1/\gamma$ becomes. Thus, from the standpoint of isolation, materials with large Verdet constants, small values of Q, and good thermal properties are preferable.

It is interesting to use our results to predict the isolation ratios for 100-W lasers such as those that will be used in the next generation of gravitational wave interferometers. Using the value $Q\alpha$ we obtained from our data, γ is expected to be 0.0015 at power levels near 100 W. This level of isolation may be insufficient for high-precision interferometric measurements and will possibly require the development of methods for minimizing these effects.

V. Conclusions

We have presented a theoretical and experimental investigation of self-induced depolarization in TGG crystals due to the absorption of laser radiation. Our theory shows that depolarization induced by the absorption of laser radiation depends upon the square of the incident power and the absorption in the material, but not on the radius of the beam. Experimental results obtained with a 9-W CW Nd : YAG laser are quantitatively consistent with the theoretical predictions

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