

Implementation of Armlocking with a delay of 1 second in the presence of Doppler shifts

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Abstract. LISA relies on several techniques to reduce the initial laser frequency noise in order to achieve an interferometric length measurement with an accuracy of $\approx 10 \text{ pm}/\sqrt{\text{Hz}}$. LISA will use ultra-stable reference cavities as a first step to reduce the laser frequency noise. In a second step the frequency will be stabilized to the LISA arms which provide a better reference in the frequency band of interest. We present experimental results demonstrating Arm locking with LISA-like light travel times and Doppler shifts. We also integrated this system with a LISA-like pre-stabilization system using our ultra-stable cavities. The addition of realistic Doppler shifts led to further refinements of the arm locking controllers compared to the controller architecture discussed in the past. A first experimental result of the new controller is also presented.

1. Introduction

The joint ESA-NASA mission LISA (Laser Interferometer Space Antenna) is being designed to observe gravitational waves in the low frequency regime from 10^{-4} to 10^{-1} Hz ([1]). LISA consists of 3 identical spacecraft (S/C) forming an equilateral triangle with an armlength of 5×10^9 m. The constellation orbits the sun trailing the earth by a distance of 50×10^9 m. Free falling test masses onboard of each spacecraft define the arms of a Michelson-like interferometer. Gravitational waves will change the lengths of the arms. These changes will be detected with a precision of about $10 \text{ pm}/\sqrt{\text{Hz}}$ by combining the individual interferometer outputs. The orbital dynamics of the constellation will cause mismatches of up to 50000 km between the arms. This mismatch requires a reduction and common mode rejection of laser frequency noise by 12 orders of magnitude with respect to its initial noise level ([2]). The current LISA baseline considers two methods to actively reduce the laser frequency noise. In a first step, the laser frequency will be stabilized to an ultra-stable optical cavity ([1],[3]). In a second step, the laser frequency will be stabilized using the LISA arms as references. This technique is called arm locking ([4]). In addition to these active frequency stabilization steps, data postprocessing algorithms (TDI - time delay interferometry ([5],[6],[7])) are used to synthesize a near equal arm interferometer to cancel the remaining laser frequency noise.

In this paper we will discuss the current status of our arm locking experiments. The motivation for arm locking is that the LISA arms provide a much better reference ($\Delta L/L = 10^{-21}$) in the LISA band compared to the ultra-stable cavities ($\Delta L/L = 10^{-13}$).

As of now, arm locking has been analyzed in several configurations ([4]) under the assumption that the Doppler shift can be subtracted from the interferometer signal. So far, experimental

demonstrations have been limited to short 'light travel times' of a few μs without any Doppler shifts ([8],[9]). In addition, the incorporation of the pre-stabilization to the optical cavity has not been demonstrated experimentally although several designs have been discussed in the past. In this paper, we report on the first experimental demonstrations of arm locking with realistic Doppler shifts, an integration with a pre-stabilization system and light travel times of one second.

2. Arm locking

In an arm locked configuration, one laser on one of the S/C will act as the master laser. Every other laser is either directly or indirectly frequency offset phase locked with a known heterodyne frequency ω_H to this laser. In this configuration, the lasers on the far S/C act as transponders for this master laser. The beat signal between the transponded field and the master laser on the master S/C is then:

$$P(t) = A \cos [(\omega_D + \omega_H)t + \varphi_1(t) - \varphi_1(t - \tau)] \quad (1)$$

where $\varphi_1(t)$ is the phase noise of the pre-stabilized laser, $\tau \approx 33\text{ s}$ is the two way propagation time between the spacecraft, and ω_D is the round-trip Doppler shift between two S/C. The phase of $P(t)$ is then measured with a phasemeter. The phasemeter consists of a numerical controlled oscillator (NCO) which is phase locked to the signal. The NCO actuator signal of the digital control loop is then used to regenerate the phase of $P(t)$ with respect to the initial frequency ω_{NCO} of the NCO:

$$S(t) = (\omega_D + \omega_H)t + \varphi_1(t) - \varphi_1(t - \tau) - \omega_{NCO}t \quad (2)$$

In an idealized arm locking system the NCO frequency can be set to $\omega_{NCO} = \omega_D + \omega_H$. Thus $S(t)$ can be written as:

$$S(t) = \varphi_1(t) - \varphi_1(t - \tau). \quad (3)$$

Laplace transforming 3, the frequency response of the interferometer is given by ([4]):

$$G(s) = \frac{S(s)}{\varphi_1(s)} = 1 - e^{-s\tau} = 2i \sin\left(\frac{\omega\tau}{2}\right) e^{\frac{-i\omega\tau}{2}}. \quad (4)$$

Figure 1 shows the Bode plot of the interferometer's transfer function for a delay of $\tau=1\text{ s}$. Thus $G(s)$ represents a transfer function of a sensor, susceptible to frequency changes and can be used for an active stabilization. Since the sensitivity rolls down with f to DC for $f < 1/\tau$, the sensor is insensitive at DC as well as at the Fourier frequencies $f_n = n/\tau$. In addition the phase changes abruptly at all f_n 's from -90° to $+90^\circ$. Consequently, the controller has to roll off with less than $1/s$ to keep sufficient phase margin close to these frequencies ([4],[10]). In our experiments, we use combinations of pole-zero pairs to approximate a $1/\sqrt{s}$ slope in the controller to maintain a phase margin of approximately 45° in the entire system. This slope limits the low frequency gain above f_1 since the system has additional bandwidth limitations caused by additional delays and phase shifts in the sensor, controller, and actuator.

Our initial arm locking experiment uses a free running voltage controlled oscillator (VCO) as laser representative which is an realistic approximation in terms of frequency noise. Figure 2 shows the general setup. The free running VCO at 6 MHz is demodulated with an 8 MHz signal generated by a NCO. The 2 MHz beat signal is then split into two parts. The first part is sent to an electronic phase delay (EPD) unit ([11]) where it is delayed by a time $\tau \approx 1\text{ s}$ and where its frequency is shifted by a Doppler frequency of -1 MHz . This delayed and frequency shifted

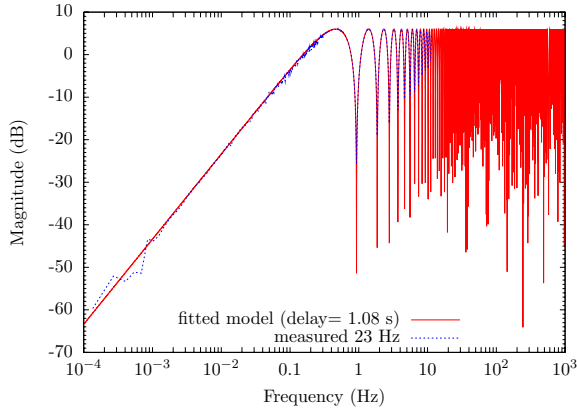


Figure 1. Transfer function of the arm locking sensor for an EPD delay of approximately $\tau=1$ s. The measured transfer function (dashed curve) is in good accordance with the theoretical model (solid).

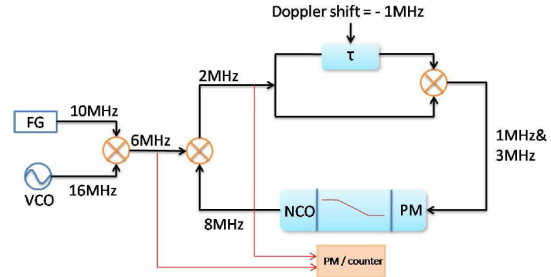


Figure 2. Setup of Armlocking implementation. The VCO signal with the frequency $\omega_0 = 6$ MHz represents a Laser beat note affected by phase noise $\varphi(t)$.

signal is then mixed in an analog mixer with the second non-delayed part of the 2 MHz signal. This is our interferometer signal which is identical to the LISA signal $S(t)$ if we lump ω_D and ω_H together into the one Doppler frequency.

An unique feature of this configuration is that it allows us to measure the noise suppression of the arm locking controller by comparing the phase noise of the VCO with the phase noise of the 2 MHz beat signal. The phase of $S(t)$ is then measured with the phasemeter (PM) ([12],[13]). The measured phase is the error signal which is then fed into the controller which then adjusts the frequency of the NCO. The closed loop will lock the difference frequency between the NCO and the VCO to the interferometer. The NCO will track the VCO frequency noise except at the frequencies where the interferometer response vanishes.

3. Arm Locking with perfect Doppler knowledge

In a first series of experiments we matched the initial NCO frequency in the arm locking PM to the Doppler shift added in the EPD unit. This corresponds to a perfect knowledge of the Doppler shift which will not be the case in LISA. The delay time in the EPD unit was set to approximately 1 s. Figure 1 shows the theoretical (solid curve) and the measured transfer function (dashed) of the arm locking sensor (interferometer). The delay time was fitted here by a least square fit estimating τ to be 1.08 s. The actuator signal to the NCO of the phasemeter was then filtered by the arm locking controller. The arm locking controller provided a slope of $1/\sqrt{f}$ at frequencies above $1/\tau$ and four integrator stages with corner frequencies at 0.1 Hz to boost the low frequency gain. This signal was then used to control the frequency of an NCO at the output of the controller.

In order to verify the performance of the arm locking loop, the initial phase/frequency noise of the VCO was measured as well as the phase/frequency noise of the stabilized beat signal. The linear spectral densities (LSD) of the VCO and the beat signal are shown in Figure 3. At frequencies given by integer multiples of the inverse delay time τ , the phase noise cannot be suppressed but is slightly enhanced (solid curve). Below about 0.1 Hz the integrators start to roll up the gain and suppress the noise further. Thus, the phase noise of the locked signal is significantly suppressed: The loop reaches zero gain¹ at a few hundred Hz (unity gain frequency)

¹ Strictly spoken the loop gain has several zero crossings (two near each spike). We refer here to the zero crossing

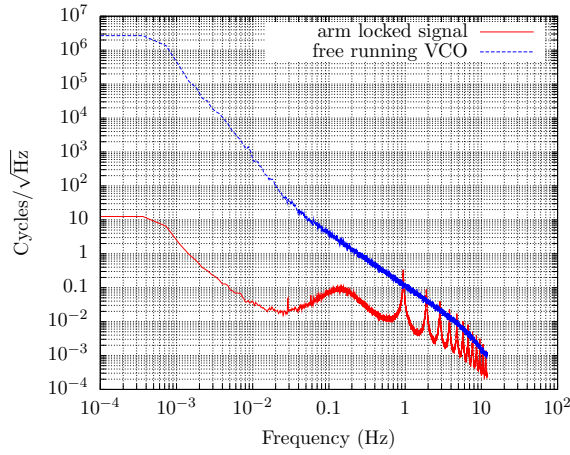


Figure 3. The free running VCO phase noise (dashed curve) and phase noise level by enabling arm locking (solid curve).

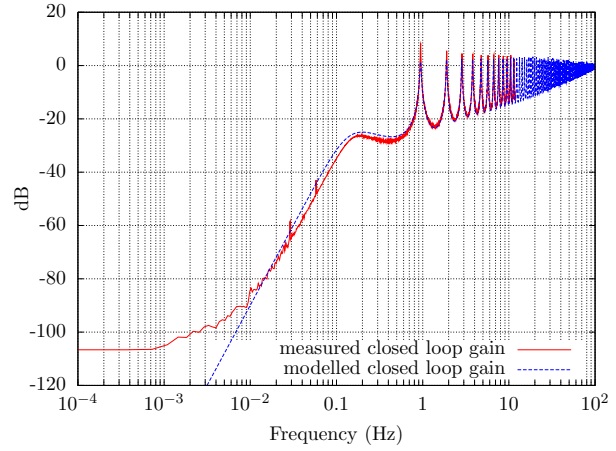


Figure 4. Modelled and measured closed loop gain (CLG).

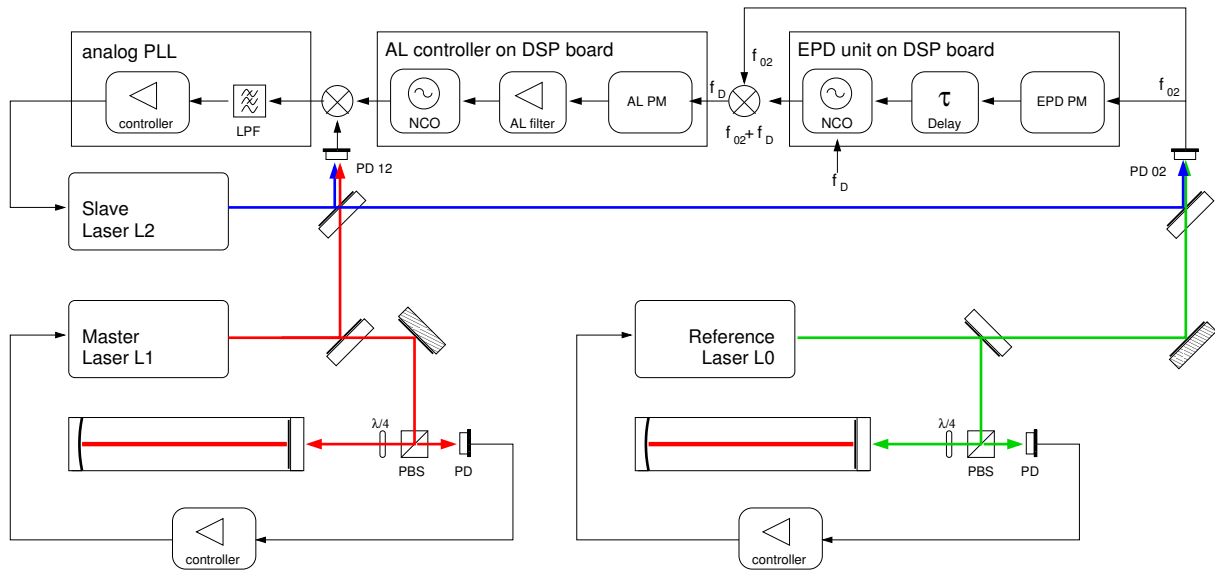


Figure 5. Electro optical setup of the laser PLL model.

rolling up to 1 Hz with $1/\sqrt{f}$ until a f^{-3} slope at approximately 0.5 Hz ensuring high gain at DC. Figure 4 shows the measured and expected closed loop gain of the system. They agree at all frequencies above about 10 mHz. The maximum noise suppression achieved is of the order of 10^5 .

4. Integration with Pre-stabilization

Figure 5 illustrates the principle of our laboratory setup of a PLL-based implementation of arm locking and pre-stabilization. The master laser (L1) is pre-stabilized to a cavity. The slave laser (L2) is offset phase locked to the master laser such that its phase noise φ_2 follows the phase noise of the master φ_1 within the bandwidth of the PLL (about 40 kHz in our setup). The offset

of the envelope of the OLG magnitude

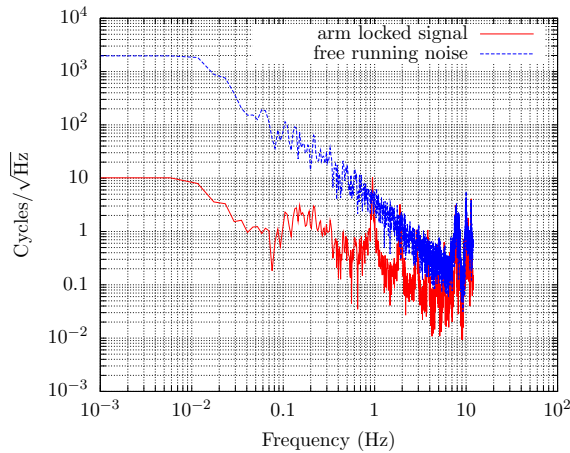


Figure 6. The upper graph is the phase noise obtained from the beat note L0-L1. The spectrum of the phase noise of the stabilized signal L0-L2 is shown as lower graph.

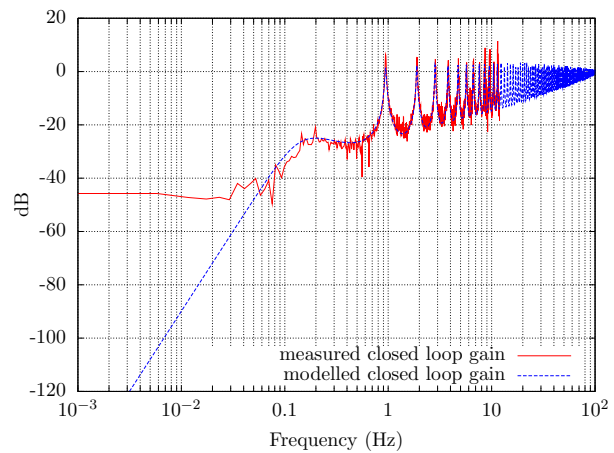


Figure 7. Modelled and measured closed loop gain. The plot shows a maximum suppression reached of about 100.

frequency is determined by a NCO which is part of the arm locking controller. The frequency of L2 can be controlled by changing the operation frequency of the NCO. In LISA, the beam of L2 propagates to the far spacecraft and the transponded copy, delayed by about 30 s, interferes with the prompt beam. Obviously, it is impossible to set up an optical delay line with a comparable delay in an optics lab. Instead of an optical delay we use an electronic delay generated by the EPD unit. In order to provide an RF signal processable for the EPD unit, L2 is mixed with an additional stabilized reference laser (L0). This signal is then electronically delayed and mixed with the instantaneous beat signal.

The obtained error signal is fed into the arm locking controller discussed above. The filtered data is used to tune the NCO to compensate frequency fluctuations of L2 with respect to L0. In other words, the phase noise φ_{02} of the beat note L0-L2 is being measured by mixing its delayed copy (1 s delay) with the prompt signal and is being suppressed by the feedback of the arm locking filter controlling the offset frequency between L2 and L1. Figure 6 shows the initial phase noise between L0 and L1 as well as the phase noise of the arm locked beat note L0-L2. The noise suppression reached at 10 mHz is about 100. Figure 7 shows the modelled and measured close loop gain of the system.

5. Conclusion and outlook

We reported on the first successful demonstration of arm locking using LISA-like signal travel times and Doppler shifts. We characterized our system first using a simplified electronic system with compensated Doppler shifts and achieved 5 orders of magnitude noise suppression in the mHz band. In addition, we also succeeded in incorporating arm locking in an existing laser pre-stabilization system. These experimental demonstrations show that single arm locking is now well understood and can be used to reduce the laser frequency noise in LISA significantly. This would further reduce the requirements on the phasemeter and TDI. We also analyzed arm locking with imperfect Doppler compensation and introduced a solution by rolling down the low frequency gain again. The experimental setup for a verification of this method is currently in implementation. Over the next months we will start to test dual arm locking, study the impact of the uncertainties and temporal changes in the Doppler shifts on dual arm locking, and integrate dual arm locking with the existing laser pre-stabilization system to demonstrate arm

locking in a full LISA-like setup.

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