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Linewidth-broadened Fabry–Perot cavities within future gravitational wave detectors

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Abstract
The bandwidth of LIGO-like terrestrial interferometric gravitational wave detectors is set by the pole of the Fabry–Perot cavities within the arms of the Michelson interferometer. This constraint arises because the gain of gravitational wave-induced signal sidebands is limited to frequencies within the linewidth of the cavities. The nature of standard Fabry–Perot cavities is such that one cannot independently adjust for increased gain without suffering a loss of bandwidth. If these quantities could be decoupled, the resulting improvement in bandwidth may lead to viable high frequency detectors. A pair of anti-parallel diffraction gratings within a Fabry–Perot cavity can increase the bandwidth of a LIGO-scale detector by a factor of ≈1000.

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1. Introduction

The quest for direct observation of gravitational waves has led scientists to construct the world's most sophisticated and sensitive interferometers. In many cases, as for example in the USA's LIGO [1] detector, the Franco–Italian VIRGO [2] detector and Japan's TAMA [3] instrument, Fabry–Perot cavities are included in the arms of a very large Michelson interferometer in order to reach these extremes of sensitivity. The cavities improve the response to gravitational wave signals within a limited low-frequency band. A signal-recycling mirror at the interferometer's anti-symmetric port, as in the GEO600 detector [10], can tune this frequency band to higher frequencies. In both cases increasing the finesse of the cavities will improve the sensitivity in an ever smaller frequency band.

Advanced LIGO [4] will have a peak sensitivity at about 300 Hz, with a $1/f$ decline in responsiveness above the peak. The anticipated sources in the several 100 Hz range are pulsars, the fundamental frequency of the intermediate phase of a neutron star–neutron star merger and the final fundamental frequency of small black hole–black hole coalescences. High frequency pulsars, the harmonics of neutron star and black-hole mergers, stellar core collapses, and neutron star oscillations, however, are expected to emit gravitational waves within the reach of advanced LIGO's sensitivity, but above its bandwidth. An instrument with at least 20 kHz of bandwidth is needed for these sources.

2. The Linewidth-enhanced cavity

2.1. The nature of the gain-bandwidth dilemma

Consider a simple Fabry–Perot cavity with arm length L and input and output mirror amplitude reflectivities and transmissivities r_1, t_1 and r_2, t_2, respectively. The normalized intensity of light of a particular frequency $\nu = c/\lambda$ within the cavity is

$$I(\nu) = \frac{t_1^2}{1 + r_1^2 r_2^2 - 2r_1 r_2 \cos(\Phi(\nu))} \tag{1}$$

with $\Phi(\nu) = 4\pi\nu L/c$, a frequency-dependent round-trip phase shift. When, as in LIGO, $r_2 \simeq 1$, and the mirror transmissivities and losses are very small, the maximum of this function can be approximated as

$$I(\nu) \approx \frac{4}{t_1^2}. \tag{2}$$

The intensity is maximum for one frequency of light (and periodically for every free spectral range thereafter) for which $\Phi(\lambda) = 2\pi n$, with n being an integer. The intracavity intensity decreases for larger or smaller frequencies at a rate determined by r_1 and r_2. The full width at half maximum (FWHM) of the Airy peaks in the intensity that corresponds to cavity resonances is FWHM $=$ FSR$/F$, with the free spectral range FSR $= c/2L$ and the finesse $F = \pi\sqrt{r_1 r_2}/(1-r_1 r_2)$. Again consider the LIGO arm cavities, with highly reflective mirrors. One finds that

$$F \approx \frac{2\pi}{t_1^2} \Longrightarrow \text{FWHM} \propto t_1^2. \tag{3}$$

It is clear that linewidth and peak light intensity are inextricably intertwined in a standard Fabry–Perot cavity. As a result, gravitational wave scientists, who would like to maximize both quantities, must compromise in choosing the cavity's parameters.

2.2. 'White-light' cavities

The round-trip phase shift's variation with frequency has been identified as the source of the gain-bandwidth dilemma, immediately suggesting it as the focus of design alterations. Perhaps Φ could be made invariant with frequency if the optical path length inside the cavity were also frequency dependent:

$$\Phi(\nu) = \frac{4\pi\nu L(\nu)}{c} = \text{constant}. \tag{4}$$

Making the constant an integer multiple of 2π ensures that light of any frequency resonates inside the altered cavity. We have thus created a 'white-light' cavity.

There are several ways in which this optical path length modification may be accomplished. The required anomalous dispersion is found in the centre of atomic absorption lines, but the absorption would generally suppress any gravitational wave-induced signals. Wicht *et al* [5] studied optically pumped atomic resonance systems and proved that one could have an appropriate anomalous dispersion for a white-light cavity at a point of vanishing absorption and optical gain.

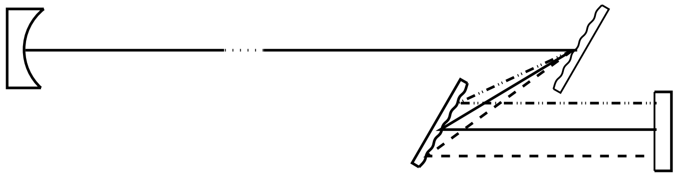

Figure 1. Grating-enhanced cavity.

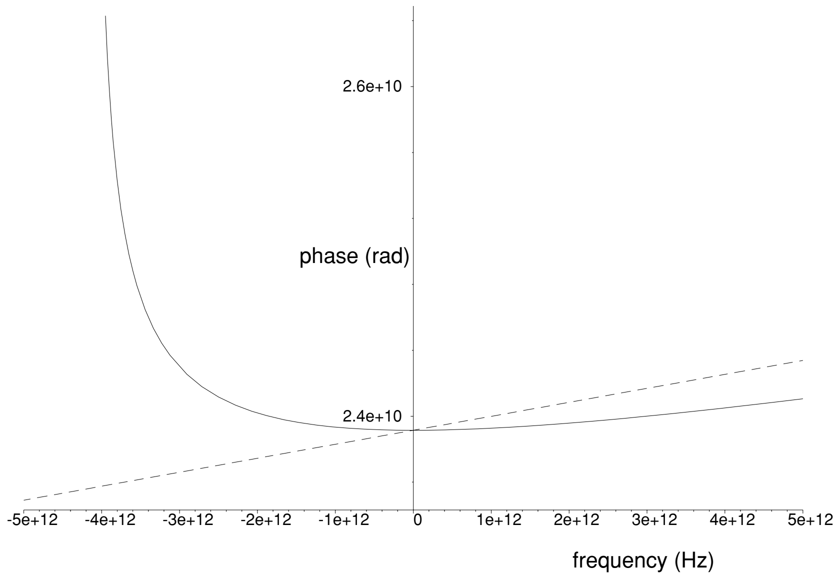

Figure 2. Comparison of round-trip phases for grating-enhanced (solid line) and standard (dashed) Fabry–Perot cavities. Frequencies on abscissa $= f - f_{\mathrm{laser}}$.

Alternatively, a pair of anti-parallel diffraction gratings placed inside the cavity creates a frequency-dependent length in a readily comprehensible way [6]. The arrangement shown in figure 1 works as follows.

(i) Monochromatic light is injected into the cavity.
(ii) A gravitational wave modulates the light-transit time between the input mirror and the distant first diffraction grating such that frequency sidebands are generated on the original laser light.
(iii) The parallel gratings allocate a different path length to light of different frequencies.
(iv) As seen in the figure, the redder lower sideband (dashed line) travels a longer distance than the bluer carrier (solid line) or upper sideband (dot-dashed line) light. This additional path can cancel the variation of the round-trip phase shift relative to the carrier light.

Figure 2 shows the resulting round-trip phase as a function of frequency and compares this to the $\Phi(\nu)$ of a standard cavity with the same nominal length. The grating-enhanced cavity will have a superior linewidth wherever its slope is less than that of the standard cavity. By adjusting such parameters as the density of rulings on the grating or the fraction of the total cavity length that is between the gratings, we can tune the interferometer to the flat minimum of the curve in figure 2, the point at which the grating-enhanced cavity's linewidth is greatest. Figure 3 shows the intensity build-up in a LIGO-scale enhanced cavity as one varies the laser

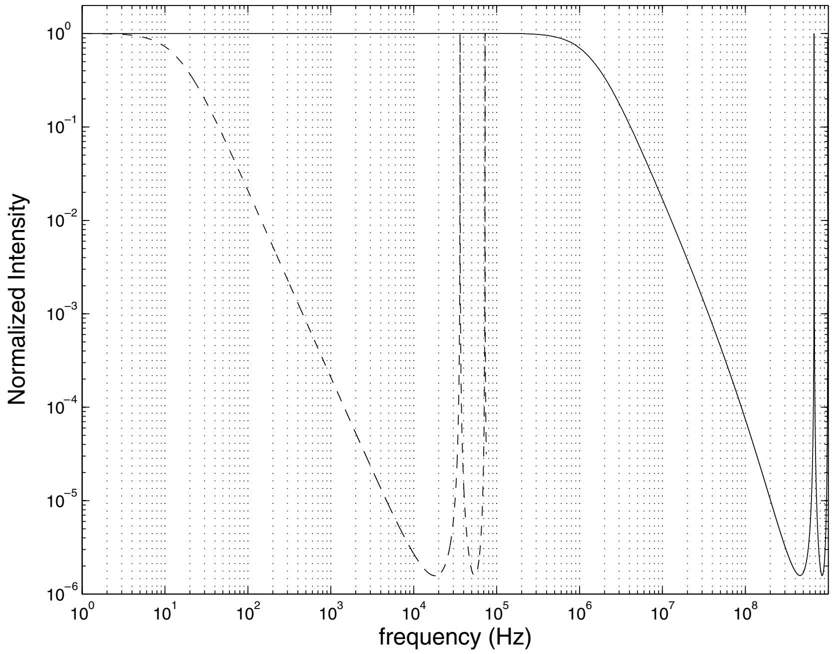

Figure 3. Linewidths of grating-enhanced (solid line) and standard (dashed line) cavitites. The standard cavity curve is cut off at 75 kHz so as not to obscure the enhanced cavity plot. Frequencies $= f - f_{\text{laser}}$.

frequency. This curve is the theoretical performance of a grating-enhanced cavity with 4134 m total one-way length, a spacing between the grating planes of 71 m, a grating constant of 1633 lines/mm, an incident angle of 54° and a laser wavelength of 1064 nm. The plot assumes the slightly idealized case of lossless gratings. Also shown is the resonance width of a standard cavity. The grating-enhanced linewidth has increased by about a million times. We note at this point, however, that this static response to changing frequency is not equivalent to gravitational wave response, as we shall discuss in the next section. The final design for a full white-light interferometer will include two grating-enhanced cavities in its arms. Because we no longer need worry about limiting our bandwidth by increasing the arm cavity mirror reflectivities, these may be matched to the grating losses for optimal coupling. Thus we may forego both power- and signal-recycling mirrors at the symmetric and anti-symmetric ports, respectively. This has the added advantage of removing power from the thick substrates of the beamsplitter and input mirrors, reducing thermal loading problems. Real gratings with losses will require greater laser power to reach the same maximum signal build-up inside the cavities as the standard cavity case. There is no theoretical limit to diffraction grating efficiency. Grating designs that use methods beyond the scalar approximation routinely produce efficiencies of essentially 100% [9]. Actual grating fabrication is rapidly catching up to theory.

3. Gravitational wave response in the time domain

In the following calculations we assume that the overall cavity length is much longer than the spacing between the gratings, and that the gratings are very near the end mirror. Gravitational wave effects need only be considered significant, therefore, as light travels between the input mirror and the first grating. The gratings and end mirror may be considered a

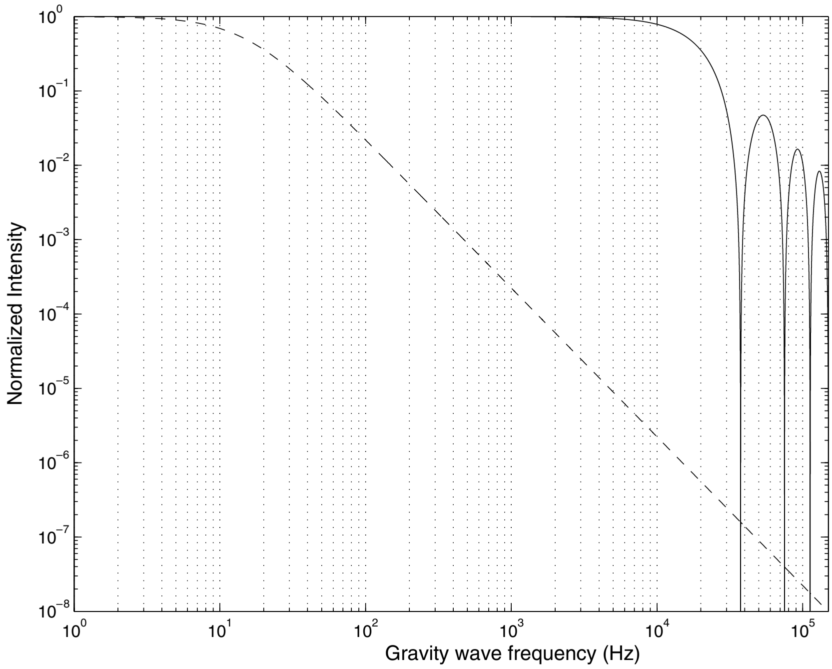

Figure 4. White-light cavity's gravitational wave response. The solid line is the normalized intracavity intensity for a white-light cavity; the dashed line is a standard cavity. Both curves have minima at multiples of the FSR, though the solid white-light curve obscures the minima of the standard cavity curve.

complex, compound mirror that imparts a frequency-dependent phase shift and time delay in reflection. The impinging gravitational wave is small and of a single frequency. Its strain is $h(t) = h_0 \cos(\Omega t)$.

The positions of the optical components are fixed while the gravitational wave is interpreted as a phase modulation of the cavity's internal laser field. The diffraction gratings (assumed perfectly efficient) will 'see' phase modulated light, and will diffract each of the three frequency components into a different path. The lower sideband, carrier and upper sideband are now also separated in time, emerging sequentially from the compound mirror, though all with the same phase. The gravitational wave's manipulation of spacetime fashions further frequency sidebands from the carrier light as it returns to the cavity input mirror, where it interferes with the incoming laser light before it propagates again through the cavity. Summing over all sidebands created in the carrier from the infinite past yields an amplitude transfer function for the upper signal sideband of the form [7]:

$$T_{\mathrm{WLC}}(\Omega) = \frac{t_1 \omega L \operatorname{sinc}(\Omega L/c) r_1 r_2 \,\mathrm{e}^{2\mathrm{i}\omega L/c}}{2c[1 - r_1 r_2 \,\mathrm{e}^{-2\mathrm{i}\omega L/c}]^2} \,\mathrm{e}^{-\mathrm{i}(\omega+\Omega)t}. \tag{5}$$

Let us compare this equation to that of a standard cavity, whose upper sideband transfer function has the form:

$$T_{\mathrm{FP}}(\Omega) = \frac{t_1 \omega L \operatorname{sinc}(\Omega L/c) r_1 r_2 \,\mathrm{e}^{2\mathrm{i}(\omega+\Omega)L/c}}{2c[1 - r_1 r_2 \,\mathrm{e}^{-2\mathrm{i}\omega L/c}][1 - r_1 r_2 \,\mathrm{e}^{-2\mathrm{i}(\omega+\Omega)L/c}]} \,\mathrm{e}^{-\mathrm{i}(\omega+\Omega)t}. \tag{6}$$

Because the cavity's length is locked to the laser, the term $\mathrm{e}^{-2\mathrm{i}\omega L/c} = 1$, hence, the $\mathrm{e}^{-2\mathrm{i}\Omega L/c}$ term in the denominator of equation (6) is the cause of the standard cavity's limited bandwidth. Figure 4 compares the sideband intensities within standard and perfect white-light cavities with identical lossless mirror reflectivities $\left(r_1^2 = 0.995, r_2 = 1\right)$ and nominal cavity lengths (4134 m). The zeros of the white-light curve, due to the sinc function in the numerator, correspond to frequencies for which the cavity round trip time causes the signal to be integrated over a full cycle. This effect is not evident in LIGO's sensitivity curves, where the arm cavity poles diminish the sensitivity well before these frequencies. We see then that the white-light cavity's true effect on the detector bandwidth will be a three-order-of-magnitude increase.

4. Conclusion

We have shown that the addition of anti-parallel gratings to a Fabry–Perot cavity can substantially enhance the bandwidth of the cavity without a corresponding reduction in gain. Both static frequency-domain and time-domain calculations have been performed which indicate that the LIGO-class interferometers can achieve high sensitivity over 20 kHz bandwidths. Current efforts are directed at developing ultra low-loss diffraction gratings and experimental implementation and characterization of white-light cavities.

Acknowledgment

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