1

Reflection and transmission at oblique incidence

The normal incidence reflectance (the ratio of the intensity of incident and reflected light) is proportional to $|\mathbf{E}_r|^2$, i. e., $\mathscr{R} = rr^*$. With $N = n + i\kappa$. The reflectance is

$$\begin{aligned} \mathscr{R} &= \left|\frac{1-N}{1+N}\right|^2 \\ &= \frac{(n-1)^2+\kappa^2}{(n+1)^2+\kappa^2}. \end{aligned} \tag{1.1}$$

The range of $\mathscr{R}$ is $0 \leftrightarrow 1$.

Now, I'll consider the reflectance at non-normal incidence. Two neat effects will appear: zero reflectance at Brewster's angle and zero transmittance in the regime of total internal reflection. I'll first discuss the relations amongst the angles of incident, reflected, and transmitted rays and then go on to obtain the transmission and reflection coefficients.

I'll work out the angle relations in two ways: first using Fermat's principle and second by the boundary conditions. Fermat's Principle of Least Time may be stated as "light travels between two given points along the path of shortest time." In a single medium, this statement is equivalent to saying that light travels in straight lines.* Fermat's

* I ignore the lensing of light by strong gravitational fields. But see Ref. 1.

principle may be regarded as an axiom, as an experimental observation, or as the results of solutions to Maxwell's equations.

Reflection and refraction

In Fig. 1.1, an incident light ray IO strikes (at point O) the interface between two media of complex refractive indices $N_a = n_a + i\kappa_a$ and $N_b = n_b + i\kappa_b$. Part of the ray is reflected as ray OR and part transmitted as ray OT. The angles that the incident, reflected, and transmitted rays make to the normals of the interface are θ_i, θ_r and θ_t, respectively.

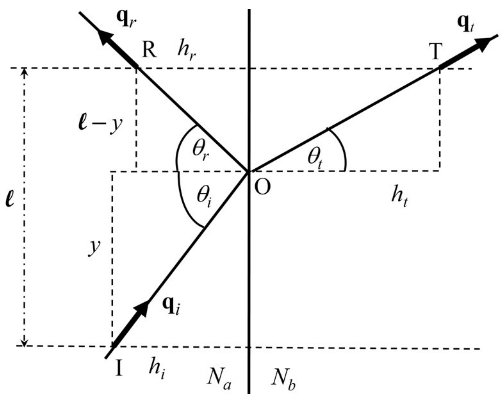

Fig. 1.1. Non-normal incidence at the interface between two media. The wave vectors $\mathbf{q}_i$, $\mathbf{q}_r$, and $\mathbf{q}_t$ for the incident, reflected and transmitted waves are shown. The points I and R are separated along the interface (vertically here) by distance ℓ and are distances h_i and h_r above the plane. The point T has the same vertical distance from O as does R, but is h_t *below* the interface.

Light in medium a travels with speed $v_a = c/n_a$. The time it takes for the light to go from point I to point R via point O is the length of the path taken divided by the speed:

$$T = T_i + T_r = \frac{\sqrt{y^2 + h_i^2}}{v_a} + \frac{\sqrt{(\ell - y)^2 + h_r^2}}{v_a}$$

with the lengths defined in Fig. 1.1. Now the light could in principal take any path that starts at I, ends at O, and includes the interface.* For instance, it could go horizontally to the right to the interface and then slant up to R, so that $y = 0$. There are an infinite number of such paths and Fermat tells me that the one taken has the minimum value of T. If so,

$$\frac{dT}{dy} = 0 = \frac{y}{v_a\sqrt{y^2 + h_i^2}} - \frac{\ell - y}{v_a\sqrt{(\ell - y)^2 + h_r^2}}.$$

I look at the triangles in the figure and see that $\sin\theta_i = y/\sqrt{y^2 + h_i^2}$ and $\sin\theta_r = (\ell - y)/\sqrt{(\ell - y)^2 + h_r^2}$. Hence, after substitution and a modest amount of algebra, I find that the angle of the light leaving the surface is the same as the arrival angle. This result is the law of reflection,†

$$\theta_i = \theta_r. \tag{1.2}$$

I drew Fig. 1.1 with points I, O, R in the plane containing also the normal to the surface. I should consider whether the light could take some skewed path and reach R by going out of the plane defined by I, O, R and the normal. But a little thought convinces me that such a path, which would go via a point on the interface O′ that is above or below point O, is longer than the one shown and so does not satisfy Fermat's Principle of Least Time. The plane containing the wave vector of the incident light ($\mathbf{q}_i$) and the surface normal is called the *plane of incidence* and the law of reflection says that this plane also contains the reflected wave vector $\mathbf{q}_r$. By the same argument, the transmitted wave vector $\mathbf{q}_t$ is also in this plane.

* Although I did not specify it, the interface must be smooth on the scale if the wavelength; this is the case of *specular* reflection. If the surface is rough, light can travel from point I to point R from many places on the interface. This second case is called diffuse reflectivity (or transmissivity) and is what you want for illumination. Metal mirrors are specular; white paper is diffuse.

† I understand the diffuse reflection from a rough surface as satisfying the law of reflection locally, but the roughness makes the local normal, when averaged over the surface, point in many of the possible directions.

When light is transmitted across the interface, its speed changes to $v_b = c/n_b$. The time taken for light to travel from I to T is

$$T = T_i + T_t = \frac{\sqrt{y^2 + h_i^2}}{v_a} + \frac{\sqrt{(\ell - y)^2 + h_t^2}}{v_b}.$$

I set the derivative to zero and note that $\sin\theta_t = (\ell - y)/\sqrt{(\ell - y)^2 + h_t^2}$. So after a modicum of algebra, including substituting for the velocities, I get Snell's law:

$$n_a \sin\theta_i = n_b \sin\theta_t. \tag{1.3}$$

Let me now consider what the boundary conditions on the fields tell me. I'll orient the coordinates so that the x axis is along the interface normal, into medium b. I'll put the y axis in the interface and in the plane of incidence and the z axis in the interface perpendicular to the plane of incidence. For these coordinates, the real parts of the three wave vectors are $\mathrm{Re}\,\mathbf{q}_i = (\omega/c)n_a(\cos\theta_i\hat{\boldsymbol{x}} + \sin\theta_i\hat{\boldsymbol{y}})$, $\mathrm{Re}\,\mathbf{q}_r = (\omega/c)n_a(-\cos\theta_r\hat{\boldsymbol{x}} + \sin\theta_r\hat{\boldsymbol{y}})$, and $\mathrm{Re}\,\mathbf{q}_t = (\omega/c)n_b(\cos\theta_t\hat{\boldsymbol{x}} + \sin\theta_t\hat{\boldsymbol{y}})$. The fields in medium a are the superposition of incident field and reflected field; in medium b the transmitted field. The boundary conditions to consider now are the continuity of tangential electric field and tangential magnetic field. These generically may be written (at $x = 0$) as

$$\mathbf{F}_i e^{i(\frac{\omega_i}{c} n_a y \sin\theta_i - \omega_i t)} + \mathbf{F}_r e^{i(\frac{\omega_r}{c} n_a y \sin\theta_r - \omega_r t)} = \mathbf{F}_t e^{i(\frac{\omega_t}{c} n_b y \sin\theta_t - \omega_t t)}, \tag{1.4}$$

where $\mathbf{F}$ is either the electric or magnetic field of the light wave and I have put subscripts on the frequency to allow me to think whether the frequencies of all three waves need to be the same.

I can address the frequency issue quickly. So that the boundary conditions remain satisfied at all times, given that they are satisfied at one instant of time, the three harmonic terms must vary in time at the same rate. This conclusion follows from the linear nature of Maxwell's equations. Hence, all fields vary in time as $e^{-i\omega t}$.

Now, the boundary conditions must be satisfied at all points on the interface as well. Suppose that they are satisfied at one particular point, such as $y = z = 0$. Then, in order that they remain true for other points on the interface, each term in Eq. 1.4 must vary in the same fashion I move along the y axis and in the plane of the interface. This requirement means that the coefficients of y must be equal. I get

$$\frac{\omega}{c} n_a \sin\theta_i = \frac{\omega}{c} n_a \sin\theta_r = \frac{\omega}{c} n_b \sin\theta_t.$$

The first pair reduce to the law of reflection $\theta_i = \theta_r$ and the first and third give Snell's law $n_a \sin\theta_i = n_b \sin\theta_t$.

Absorbing media

The propagation in strongly absorbing materials of waves incident at oblique angles to interfaces is complicated. Let me pose the problem and discuss it qualitatively. For the moment, I'll take the material in which the incident and reflected waves are traveling to be nonabsorbing.*

The geometry is shown in Fig. 1.2. The incident wave arrives at the interface at an angle to the normal. The planes of constant phase, to which the incident wave vector $\mathbf{q}_i$ are normal, are also inclined. Because medium a is nonabsorbing, the intensity at the interface (or just before the interface) is everywhere the same.

Light reaching the interface is refracted to continue as the transmitted wave. The planes of constant phase for the refracted wave, perpendicular to $\mathbf{q}_t$, are also inclined to the interface.

What about the planes of constant amplitude? If the fields were to be written as $e^{i\mathbf{q}_t\cdot\mathbf{r}-\omega t}$, the planes of constant amplitude would also be orthogonal to $\mathbf{q}_t$. But this claim is not correct. I see where the misconception comes from by looking at the four rays shown as dashed

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* A semi-infinite absorbing medium poses conceptual dissonance. For a finite amount of energy to arrive at the interface, it must have started with infinite energy.

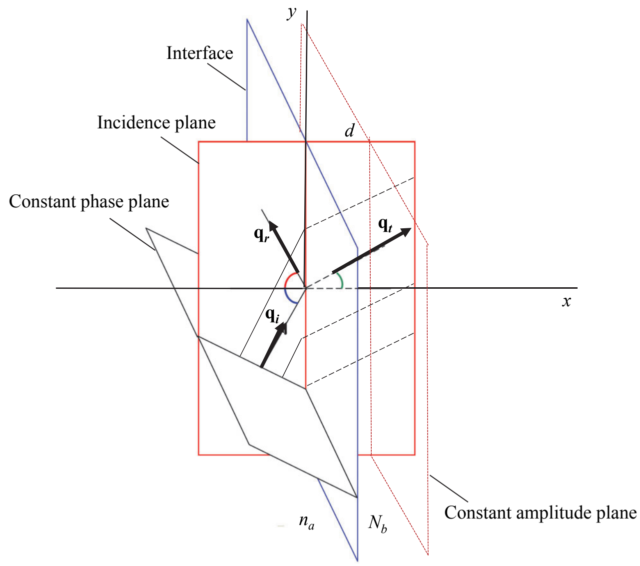

Fig. 1.2. Non-normal incidence at the interface between two media, one with refractive index n_a and the other with complex index N_b. The wave vectors for the incident, reflected and transmitted waves are shown. Also shown are the plane of incidence, the plane of constant phase for the incident wave, and the plane of constant amplitude for the transmitted wave.

lines in Fig. 1.2. When they arrive at a plane perpendicular to $\mathbf{q}_t$, the rays will have gone different distances in the absorbing medium, with the one at the bottom of the diagram having gone the farthest and the one at the top the shortest distance. Now, after crossing the interface, each ray or pencil of light does follow $E{\sim}e^{-\kappa_b \omega r/c}$ and, after reaching a depth d in medium b, has amplitude $e^{-\kappa_b \omega d/c \cos\theta_t}$. The planes of constant amplitude are parallel to the interface, as indicated in the figure. A wave where the planes of constant phase and the planes of constant amplitude are not the same is an *inhomogeneous wave*.[2–4] The details are worked out in Born and Wolf.[2] To evade this complication, I will take the refractive indices as real. The interesting phenomena and physics are in transparent media anyway.

For further ease, I'll take the materials to have $\mu_r = 1$. The equations with different values of permeability may be seen in Jackson.[5]

Amplitude equations

It turns out that the oblique-angle reflection depends both on the angle of incidence of the light and on its polarization (**E**-vector direction). The equations are called the "Fresnel equations" and it is conventional (and correct) to work them out separately for the electric field parallel and perpendicular to the plane of incidence. Any other polarization direction can be worked out by superposition.

The terminology is diverse and often used without explanation. If the electric field lies in (is parallel to) the plane of incidence, I'll call it p polarization, for "parallel." The English word for perpendicular starts with same letter* so it is termed s polarization (from "senkrecht," German for perpendicular). Other terms used for p polarization are transverse-magnetic (TM), pi-polarized, π-polarized, tangential-plane-polarized, and $\|$-polarized; s polarization can be called transverse-electric (TE), sigma-polarized, σ-polarized, sagittal-plane-polarized, or $\perp$-polarized. Finally, the plane of incidence, to which the field is either perpendicular or parallel, is the plane defined by the normal to the flat interface and the incident wave vector, $\mathbf{q}_i$.† The boundary conditions are that the normal components of **D** and **B** and the tangential components of **E** and **H** are continuous across the interface. (With $\mu_r = 1$, $\mathbf{B} = \mu_0\mathbf{H}$.) I'll use the coordinate system of Fig. 1.2, with the x axis perpendicular to the interface. The y and z directions lie in the interface, with y also in the plane of incidence and z perpendicular to it.

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* One source of confusion.

† It is *not* the plane of the interface. Also, normal incidence has no plane of incidence (or has all planes as the plane of incidence) and the subscripts p and s need not be used.

Electric field perpendicular to the plane of incidence (s)

The coordinate system and field directions are shown in Fig. 1.3. I write the fields (at the origin and at $t = 0$) as

$$\begin{aligned} \mathbf{E}_i &= \hat{\boldsymbol{z}} \\ \mathbf{E}_r &= r_s \hat{\boldsymbol{z}} \\ \mathbf{E}_t &= t_s \hat{\boldsymbol{z}} \end{aligned} \tag{1.5}$$

$$\begin{aligned} \mathbf{B}_i &= n_a(\hat{\boldsymbol{x}} \sin\theta_i - \hat{\boldsymbol{y}} \cos\theta_i) \\ \mathbf{B}_r &= n_a r_s(\hat{\boldsymbol{x}} \sin\theta_i + \hat{\boldsymbol{y}} \cos\theta_i) \\ \mathbf{B}_t &= n_b t_s(\hat{\boldsymbol{x}} \sin\theta_t - \hat{\boldsymbol{y}} \cos\theta_t) \end{aligned} \tag{1.5}$$

where I took the amplitude of the incident field as unity, r_s is the amplitude of the reflected field, and t_s is the amplitude of the transmitted field. For the magnetic fields, I used $\hat{\boldsymbol{q}} \times \hat{\boldsymbol{e}}$ for the direction and Eq. `<Hinside>` ($H = NE$) for the magnitudes.

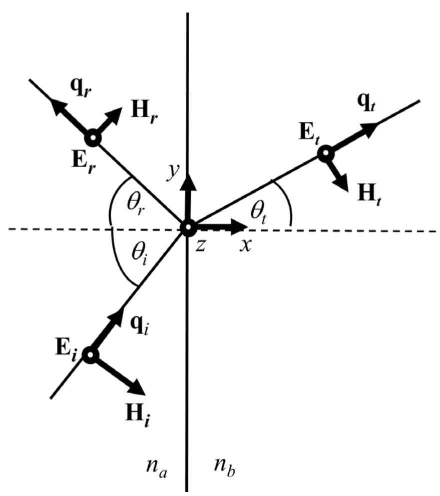

Fig. 1.3. Geometry of transmission and reflection for s-polarized light. The interface between the two media (with refractive indices n_a and n_b) is the y-z plane. The plane of the paper is the plane of incidence.

Now, I apply the boundary conditions. Tangential $\mathbf{E}$ is continuous.

I dot the $\mathbf{E}$ field with $\hat{\boldsymbol{z}}$ and equate the fields on left and right side of the interface:

$$1 + r_s = t_s, \tag{1.6}$$

because the electric field is purely tangential. Tangential $\mathbf{B}$ is continuous. The magnetic field has both tangential and normal components. For the former, I dot with $\hat{\boldsymbol{y}}$ yielding

$$-n_a \cos\theta_i + n_a r_s \cos\theta_i = -n_b t_s \cos\theta_t. \tag{1.7}$$

I substitute t_s from Eq. 1.6 into Eq. 1.7, and solve for the reflectivity

$$r_s = \frac{n_a \cos\theta_i - n_b \cos\theta_t}{n_a \cos\theta_i + n_b \cos\theta_t}. \tag{1.8}$$

Now, I put r_s from Eq. 1.8 back into Eq. 1.6. It cleans up nicely:

$$t_s = \frac{2n_a \cos\theta_i}{n_a \cos\theta_i + n_b \cos\theta_t} \tag{1.9}$$

The angle $\theta_i = 0$ at normal incidence as is $\theta_t = 0$, by Snell's law. Equations 1.8 and 1.9 reduce to the equations for normal incidence as they should. The other limit, $\theta_i = 90°$, requires* me to specify that $n_a < n_b$, in which case $r_s = 1$ and $t_s = 0$.

I used two of the four boundary conditions. What about the other two. Well, $\mathbf{D} \cdot \hat{\boldsymbol{x}} = 0$ for all three fields, so the normal $\mathbf{D}$ condition is trivially satisfied. As to normal $\mathbf{B}$, I use $\mu_r = 1$ for both materials, so taking the dot product of the magnetic fields in Eq. 1.5 with $\hat{\boldsymbol{x}}$ gives

$$n_a \sin\theta_i + r_s n_a \sin\theta_i = t_s n_b \sin\theta_t.$$

But, Eq. 1.6 gives $1 + r_s = t_s$. By use of this boundary condition, I rederive Snell's law.

* Otherwise Snell's law requires an angle for which the sine is greater than 1.

Electric field parallel to the plane of incidence (p)

The coordinate system and field directions are shown in Fig. 1.4. I've set the field directions so that the case where $\theta_i = \theta_r = 0$ in Fig. 1.4 uses the same convention as in Fig. 1.5: the electric fields are parallel and the magnetic fields antiparallel. I should warn you that about half the time I see a different convention. Some books[6,7] use this convention; others[2,5,8] use the opposite one.

I write the fields (at the origin and at $t = 0$) as

$$\begin{aligned}
\mathbf{E}_i &= -\hat{\boldsymbol{x}}\sin\theta_i + \hat{\boldsymbol{y}}\cos\theta_i \\
\mathbf{E}_r &= r_p(\hat{\boldsymbol{x}}\sin\theta_i + \hat{\boldsymbol{y}}\cos\theta_i) \\
\mathbf{E}_t &= t_p(-\hat{\boldsymbol{x}}\sin\theta_t + \hat{\boldsymbol{y}}\cos\theta_t) \\
\mathbf{B}_i &= n_a\hat{\boldsymbol{z}} \\
\mathbf{B}_r &= -n_a r_p\hat{\boldsymbol{z}} \\
\mathbf{B}_t &= n_b t_p\hat{\boldsymbol{z}}
\end{aligned} \tag{1.10}$$

where I took the amplitude of the incident field as unity, r_p is the amplitude of the reflected field, and t_p is the amplitude of the transmitted field. I used $\hat{\boldsymbol{q}} \times \hat{\boldsymbol{e}}$ for the direction of the magnetic fields and $(B = NE/c)$ for their magnitudes.

Now, I apply the boundary conditions. Tangential $\mathbf{E}$ is continuous. I dot the $\mathbf{E}$ field with $\hat{\boldsymbol{y}}$ and equate the fields on left and right side of the interface:

$$\cos\theta_i + r_p\cos\theta_i = t_p\cos\theta_t. \tag{1.11}$$

Tangential $\mathbf{H}$ $(= \mathbf{B}\mu_0)$ is continuous. The magnetic field has only tangential components. I dot with $\hat{\boldsymbol{z}}$ yielding

$$n_a - n_a r_p = n_b t_p. \tag{1.12}$$

I substitute t_p from Eq. 1.12 into Eq. 1.11, and solve for the reflectivity

$$r_p = \frac{n_a\cos\theta_t - n_b\cos\theta_i}{n_a\cos\theta_t + n_b\cos\theta_i} \tag{1.13}$$

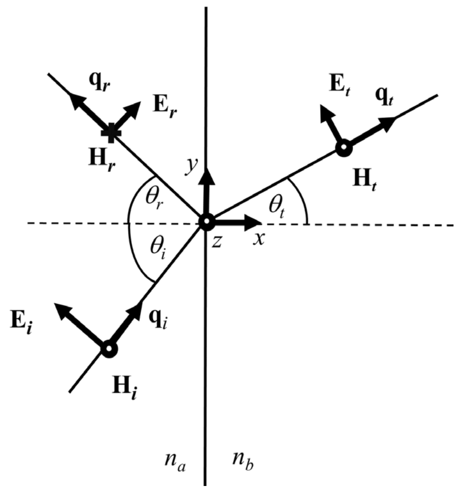

Fig. 1.4. Geometry of transmission and reflection for p-polarized light. The interface between the two media (with refractive indices n_a and n_b) is the y-z plane.

Now, I put r_p from Eq. 1.13 back into Eq. 1.12. It cleans up nicely:

$$t_p = \frac{2n_a \cos\theta_i}{n_a \cos\theta_t + n_b \cos\theta_i} \tag{1.14}$$

Eqs. 1.13 and 1.14 reduce to Eqs. `<rab>` and `<tab>` at normal incidence, as they should. The other limit, $\theta_i = 90°$, again requires me to specify that $n_a < n_b$, in which case $r_p = 1$ and $t_p = 0$.

As far as the other two boundary conditions go, the condition on $\mathbf{B}_n$ is trivially satisfied and the condition on $\mathbf{D}_n$ gives Snell's law.

Equations 1.8, 1.9, 1.13, and 1.14 all contain both $\cos\theta_i$ and $\cos\theta_t$. These quantities are not independent; Snell's law ($n_a \sin\theta_i = n_b \sin\theta_t$) relates them. When I need to, I can use

$$\cos\theta_t = \sqrt{1 - \frac{n_a^2}{n_b^2}\sin^2\theta_i} \tag{1.15}$$

to eliminate $\cos\theta_t$. Meantime, however, I'll calculate the reflected amplitudes in 4 cases: both polarizations with $n_b > n_a$ and both polarizations with $n_b < n_a$. The results are in Fig. 1.6.

The left panel shows the results for the case where the light is incident from the "less dense" (smaller n) material onto the "more

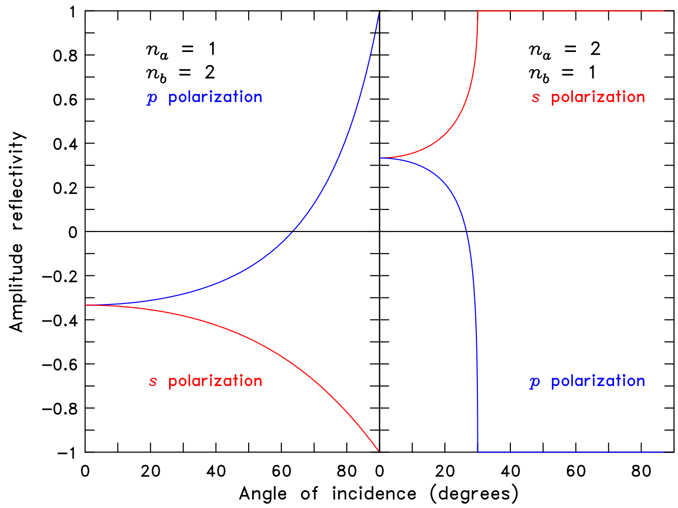

Fig. 1.6. Amplitude reflectivity for p- and s-polarized light as a function of angle of incidence. Left panel: The media have $n_a = 1$ and $n_b = 2$. Right panel: The media have $n_a = 2$ and $n_b = 1$.

dense" (larger n) material. At normal incidence $\theta_i = 0$, the reflectivity is negative, $r = -1/3$ (for $n_a = 1$ and $n_b = 2$) in both the s and p cases* The reflectivity is basically flat for the first 5° or so but the two curves separate at larger angles. The s-polarized reflectivity becomes more negative (and hence larger in magnitude) eventually reaching $r_s = -1$ at 90° incident angle. The p-polarized reflectivity becomes smaller in magnitude and crosses zero at an incident angle (for the refractive indices used) just above 60°. Then it grows in magnitude, reaching $r_p = +1$ at 90° incident angle. Thus both polarizations have identical reflectivities at normal and at grazing incidence but nowhere in between.

There are differences and similarities when the light is incident from the "more dense" (larger n) material onto the "less dense" (smaller n) material. The normal incidence reflectivity is positive rather than negative, with the same magnitude as the case I discussed above. The p-polarized reflectivity crosses zero at a smaller incident angle, just

* As it should be. There is no plane of incidence when $\theta_i = 0$. (Or,there is an infinite number of them.)

above 25° for the parameters used. The reflectivity reaches ±1, but at angles well below grazing incidence. Both s-polarized and p-polarized reflectivities reach $r_s = +1$ and $r_p = -1$ at 30° incident angle.

Special angles

There is one angle for a given n_a and n_b when the value of r_p goes to zero.* All the p-polarized incident light is purely refracted into the second medium. To find the angle, I set $r_p = 0$ in Eq. 1.13 to find

$$n_b \cos\theta_i = n_a \cos\theta_t = n_a \sqrt{1 - \frac{n_a^2}{n_b^2}\sin^2\theta_i},$$

where I've used Eq. 1.15 on the right to eliminate θ_t. I square both sides, use $1 = \sin^2\theta_i + \cos^2\theta_i$, collect terms, take a square root, use $\tan = \sin/\cos$, and find that the angle of zero reflectivity, θ_B obeys

$$\tan\theta_B = \frac{n_b}{n_a}. \tag{1.16}$$

This angle is known as Brewster's angle. It is 63.4° for the left panel of Fig. 1.6 and 26.6° for the right panel. It is 56° for an interface between glass ($n = 1.5$) and vacuum or air ($n = 1$). Finally for a silicon ($n = 3.48$)/vacuum interface, $\theta_B = 74°$.

When I calculate the angle between the reflected wave and the transmitted wave (the angle $\pi - \theta_r - \theta_t$ in Fig. 1.4 when $\theta_r = \theta_i = \theta_B$), I find that the angle is 90°. I can see this in several ways: I can compute the angle using Eqs. 1.3 and 1.15 as well as some trigonometric identities and find that the angle is $\pi/2$. An alternate way is to compute the angle for which $\mathbf{q}_r \cdot \mathbf{q}_t = 0$ and find that it is indeed θ_B.

There is some physics in the fact that the reflectance is zero when the reflected and transmitted rays are at right angles to each other.

* The angle at which r_p changes sign! This statement is true for nonabsorbing media. In contrast, if there is absorption, there is a complex refractive index and the amplitude reflectivity is complex in turn. The magnitude of the complex reflectivity is always positive. It can get small at some angle, but will not be zero.

I can ask "why is there light reflected from a surface and what are the sources of this light?" Electromagnetic radiation has as its sources accelerated charges. The transmitted field in medium b induces a time-varying electric dipole moment/unit volume $\mathbf{P}_b = \chi_b \mathbf{E}_t = (\epsilon_b - 1)\mathbf{E}_t/4\pi$ where χ_b is the complex susceptibility and ϵ_b is the complex dielectric function. The dipole moment/unit volume is composed of a very large number of oscillating dipoles, each of which emits dipole radiation. The reflected light is the superposition of the emitted fields from all these dipoles. The dipoles are oriented along $\mathbf{E}_t$, which is perpendicular to $\mathbf{q}_t$ and hence is, for p-polarized light, parallel to $\mathbf{q}_r$. But the radiated power is zero along the axis of the dipole; hence, the reflectivity is zero.

At Brewster's angle, reflected light is 100% s-polarized and transmitted light is preferentially p-polarized. A variety of devices take advantage of this selectivity. One of the simplest is a polarizer oriented for p polarization, and therefore suppressing reflected light from windows in photographs or optical instruments. A "pile of plates," at Brewster's angle, each reflecting a few percent of the s-polarized light, may be used to provide highly polarized transmitted light over a broad range of wavelengths.

The other interesting angle, known as the *critical angle*, appears in the right panel of Fig. 1.6; it is the angle of incidence at and beyond which the reflectivity is ± 1. The phenomenon is known as "total internal reflection." Whereas Brewster's angle occurs only for p-polarized light and is found independent of which index is larger, the critical angle is found only if $n_a > n_b$ and occurs for both polarizations.

Snell's law, Eq. 1.3, can be written $\sin\theta_t = (n_a/n_b)\sin\theta_i$. If $n_a > n_b$, then $\theta_t > \theta_i$. There is then some incident angle for which $\theta_t = 90°$. This, the critical angle is then given by

$$\theta_c = \arcsin\left(\frac{n_b}{n_a}\right).$$

All light is reflected and $|r_s| = |r_p| = 1$. The critical angle is 41.8° for a glass ($n_a = 1.5$) to air or vacuum ($n_b = 1$) interface. For silicon

($n = 3.48$) to vacuum, $\theta_c = 16.7°$. Recall that the angles are measured with respect to the normal, so light does not need to be sent to the interface at near grazing incidence to have total internal reflection.

The effect of total internal reflection is to contain the light inside the medium. It is the basis of fiber optics and other dielectric wave guides or light pipes.

That there is no transmitted light does not mean that the fields are zero in medium b. In fact, there are exponentially decaying evanescent fields in the second medium.[2,5]

1.1 Reflectance and Transmittance

The Poynting vectors of incident, reflected, and transmitted waves tell me the intensities of the light in these beams. But the reflectance $\mathscr{R}$ and transmittance $\mathscr{T}$ represent the fraction of the incident energy that is reflected or transmitted by the interface. Because intensity is energy/area, I'll have to consider the change of area of the beams.

For reflectance it is easy: the incident and reflected waves propagate in the same medium and make the same angle with the normal to the surface. Consequently the reflectance is

$$\mathscr{R} = |r|^2 \, .$$

The transmittance $\mathscr{T}$ is generally not equal to $|t|^2$ for two reasons. First, the Poynting vector includes a factor of the refractive index n, just at it did in the case of normal incidence on p. `<Poynting>`. Second, because the light is refracted into a different direction, the cross-sectional areas of a pencil or rays is different in the two media. The transmittance is

$$\mathscr{T} = \frac{n_b \cos\theta_t}{n_a \cos\theta_i} \, |t|^2 \, . \tag{1.17}$$

The factor of n_b/n_a comes from the ratio of Poynting vectors. The factor of $\cos\theta_t / \cos\theta_i$ represents the change in area of the pencil of rays.

The reflectance for s-polarized light is

$$R_s = \left|\frac{n_a\cos\theta_i - n_b\cos\theta_t}{n_a\cos\theta_i + n_b\cos\theta_t}\right|^2 = \left|\frac{n_a\cos\theta_i - n_b\sqrt{1-\frac{n_a^2}{n_b^2}\sin^2\theta_i}}{n_a\cos\theta_i + n_b\sqrt{1-\frac{n_a^2}{n_b^2}\sin^2\theta_i}}\right|^2,$$

while the reflectance for p-polarized light is

$$R_p = \left|\frac{n_a\cos\theta_t - n_b\cos\theta_i}{n_a\cos\theta_t + n_b\cos\theta_i}\right|^2 = \left|\frac{n_a\sqrt{1-\frac{n_a^2}{n_b^2}\sin^2\theta_i} - n_b\cos\theta_i}{n_a\sqrt{1-\frac{n_a^2}{n_b^2}\sin^2\theta_i} + n_b\cos\theta_i}\right|^2.$$

To obtain the second form of each equation I eliminated $\cos\theta_t$ using Eq. 1.15.

I can calculate the transmittances for the two polarizations from Eqs. 1.17, 1.9, and 1.14. As an alternative, I can invoke the conservation of energy and then the transmission coefficients are given by

$$\mathscr{T}_s = 1 - \mathscr{R}_s, \tag{1.18}$$

and

$$\mathscr{T}_p = 1 - \mathscr{R}_p. \tag{1.19}$$

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