How to run the sign correlation analysis in LDAS?

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Abbreviations:

- **XC – cross-check, which allows to confirm validity of the data and the data analysis.**

Cross-Correlation in Wavelet Domain

- **x-correlation is a sum over wavelet layers**
  - τ **– time lag**
  - n **– wavelet layer number**
  - N_k **– number of samples in layer k**
  - $r_k(\tau)$ **– correlation coefficients as a function of lag time t**

$$S = \sum_{n,\tau} N_k w_k(\tau) r_k(\tau)$$

- $w_k(\tau)$ **– optimal weight**

$$w_k(\tau) = \int_{-\infty}^{\infty} df \left|\psi_k(f)\right|^2 |f|^{-3} \frac{\Omega_{GW}(f)\cdot\gamma(f,\Omega_L,\Omega_H)}{R_L(f)P_L(f)/\sigma_k^L \cdot R_H(f)P_H(f)/\sigma_k^H} \exp(-j2\pi f\tau)$$

- ✓ Ψ_k **– Fourier image of mother wavelet for layer k**
- ✓ γ **– overlap reduction function**
- ✓ σ_n^L, σ_n^H **- noise *rms* in wavelet domain for detector L (H)**
- ✓ ***R(f)*** **– detector responses**

Sign Cross-Correlation

- **Sign x-correlation**

$$S_s = \sum_{n,\tau} N_k \omega_k(\tau) \rho_k(\tau)$$

  - **$\rho_k(\tau)$ – sign correlation coefficients**
  - **$\omega_k(\tau)$ – optimal filter**

- **optimal filter**

$$\omega_k(\tau) = \frac{\varepsilon_k \cdot w_k(\tau)}{v_k(\tau)}$$

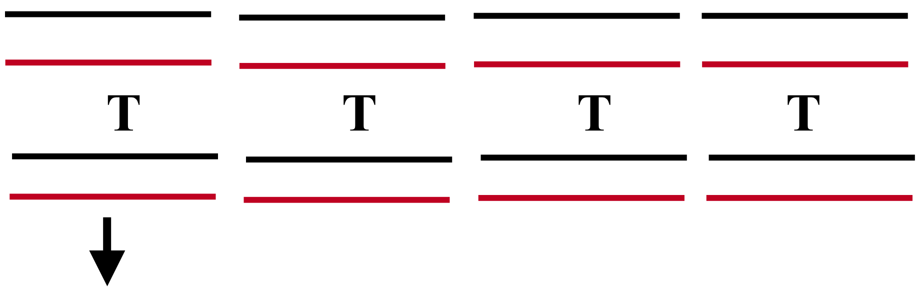
  - **ε_k – sign correlation efficiency**
  - **$v_k(\tau)$ – contribution from correlated noise**
  - $\varepsilon_k \cdot w_k(\tau)$ **should not vary with time**

- **Variance of ρ**

$$\mathrm{var}(\rho_k(\tau)) = \frac{1}{N_k} v_k(\tau)$$

What we can calculate with LDAS?

Process data segments T sec long (T=1024sec)



x_h - data
$h_h(\Omega)$ - simulation

x_l - data
$h_l(\Omega)$ - simulation

$\rho_\kappa(\tau)$ ------ *- sign correlation coefficient for layer k and lag* τ

$v_\kappa(\tau)$ ------ *- variance of the sign correlation coefficients*

$\sigma^L{}_k$ ------ *- rms of wavelet coefficients for IFO L*

$\sigma^H{}_k$ ------ *- rms of wavelet coefficients for IFO H*

$W_\kappa(\tau)$ ------ *- optimal coefficients (output of DSO)*

$r_\kappa(\tau,\Omega)$ ------ *- linear correlation coefficients for* $(x_h+h_h)*(x_l+h_l)$

$\rho_\kappa(\tau,\Omega)$ ------ *- sign correlation coefficients* $(x_h+h_h)*(x_l+h_l)$

What we calculate in datacondAPI?

1. ***Do wavelet transform – WaveletForward()***
2. ***Extract wavelet layer – GetLayer(x,k)***
3. ***Calculate rms of wavelet coefficients*** σ^I_k ***(I=H,L)***
4. ***Convert to sign time series – u=signum(x)***
5. ***Calculate sign x-statistics –*** $s=mul(u_H,u_L)$
6. ***Calculate x-correlation coefficient -*** $\rho_k(\tau)=mean(s)$
7. Calculate ρ variance
   - psd = mul(fft(s), conj(fft(s))) – need to be fixed
   - auto-correlation function of s - $a(\tau) = fft^{-1}(psd)$
   - calculate and save ***v(k)*** – estimate of correlated noise

What we get from ν?

$$\nu = 1 + \sum_{m=1}^{T_s/\Delta t} (n-m) a_n(m\Delta t) = \mathrm{var}_c(\rho\sqrt{n})$$

- ν takes in to account the second-order statistics $P(s_i,s_j)$
  - s – sign cross-correlation statistics
- ν depends on
  - a_n – correlated noise autocorrelation function (estimated from s)
  - T_s - correlation time scale
- ν is a measure of correlated noise, or quality of data.
  - XC: look at ν value and its variation with time.
  - XC: check contribution of correlated noise at different time scales T_s
- ν is used for calculation of the variance of ρ

How to calculate ε_k?

- ε_k is the efficiency of the sign x-correlation
- ε_k does not depend on
  - SGW model and strength of SGW signal
  - detector response
  - overlap reduction function
- Therefore ε_k can be calculated by adding a white Gaussian noise to the output of the interferometers, the same both for L and H. Then by measuring the sign and linear correlation coefficients we can find ε_k as

$$\varepsilon_k = \frac{\rho_k(\tau,\Omega)}{r_k(\tau,\Omega)}$$

How to calculate optimal filter?

- **Optimal filter**

$$\omega_k(\tau) = \varepsilon_k \cdot \frac{w_k(\tau)}{v_k(\tau)} = \varepsilon_k \cdot \frac{\sigma_k^H \sigma_k^L W_k(\tau)}{v_k(\tau)}$$

  - **where $W_k(\tau)$ is the output of stochastic DSO**

$$W_k(\tau) = \int_{-\infty}^{\infty} df \left|\psi_k(f)\right|^2 \left|f\right|^{-3} \frac{\Omega_{GW}(f) \cdot \gamma(f, \Omega_L, \Omega_H)}{R_L(f)P_L(f) \cdot R_H(f)P_H(f)} \exp(-j2\pi f\tau)$$

    - ✓ **just input $\Psi_k(t)$ and $\Psi_k(t+\tau)$ instead of the interferometer data**

- **XC: optimal filter should not vary with time if the detector calibration is correct**

Upper Limit

- **Cross-correlation & variance:**

$$S_s = \sum_{n,\tau} N_k \omega_k(\tau) \rho_k(\tau) \qquad V_s = \sum_{n,\tau} N_k \omega_k^2(\tau) v_k(\tau)$$

- **x-correlation expectation value:** $$\mu = \Omega \sum_{n,\tau} N_k \omega_k^2(\tau) v_k(\tau) = \Omega V_s$$

- **signal to noise ratio:** $$SNR = \Omega \sqrt{\sum_{n,\tau} N_k \omega_k^2(\tau) v_k(\tau)} = \Omega \sqrt{V_s}$$

- **confidence level:** $$CL = \frac{1}{2} erf\left(\frac{S_s}{\sqrt{V_s}}\right) \rightarrow \tilde{S}_s(95CL)$$

- **upper limit:** $$\tilde{\Omega} = \frac{\tilde{S}_s(95CL)}{V_s}$$