Physics demonstrations and experiments which can be set up with minimal equipment and at almost no cost.

Hole-punching paper for physics and fun

A two-dimensional percolation exercise

A diverse collection of phenomena in physics, chemistry and biology may be described in terms of percolation theory, and some examples include the nature of alloys [1] and the robustness of the Internet [2]. Although some experiments [3, 4] and simulations [5] have been described and are available, most of these possibilities are too complicated, time-consuming or expensive for a typical high school laboratory. Herein, we describe a simple exercise that, to our knowledge, has not been previously reported. This investigation has been successfully implemented into the laboratory exercises for high school students.

Background

As a simple case, consider a sheet of aluminium foil with two electrodes attached at the ends and connected to multimeter that can report the resistance. In this initial condition, current will easily flow, and the resistance is low. Now reconsider this situation when holes are randomly punched into the foil. When there are only a few holes in the sheet, the resistance should be close to its initial value. However, when the number of holes approaches the point when there will be no path for the current to flow across the sheet, the resistance will increase, eventually diverging to the resistance value of the open circuit. This arrangement is an example of a two-dimensional percolation problem, and the resistance diverges at a critical point p_C, which is the ratio of the area of the aluminium that must be removed to the initial total area of the foil. This value of p_C is the critical point of this interesting phase transition between the conducting and insulating states.

To avoid the need to measure the resistance of aluminium foil, we have generated a mechanical analogue of this problem. Specifically, a small sheet of paper is suspended from a fixed clamp and a second clamp acts as an added weight at the bottom of the sheet. Holes are randomly punched in the paper until the weight falls, representing the phase transition.

Materials

- 1 – sheet 4 squares per inch graph paper
- 1 – 1/4 inch (6 mm) hole punch with a 1 inch (25 mm) throat
- 2 – heavy duty paper clamps
- 1 – 12 sided die
- 1 – 10 sided die

To reduce cost, the 10 and 12 sided dice can be substituted with another suitable random number generator. Sets of numbered bingo chips and paper decahedrons and dodecahedrons [6] have been used

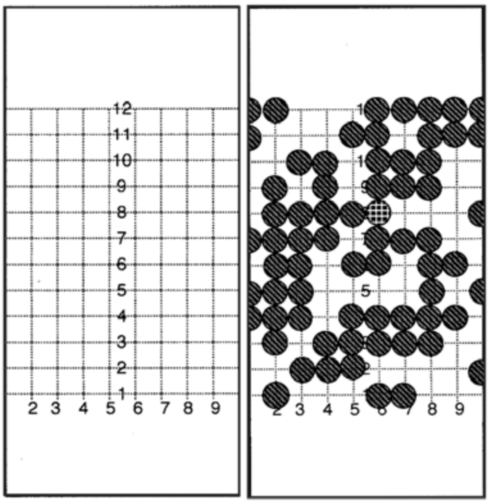

Figure 1. *The initial strip of graph paper is shown (left). A typical punched strip (right) shows the last hole to be punched (6,8) when the critical point has been achieved and the paper separates into pieces.*

successfully. The construction of the figures can add to the preparation time but may be a positive exercise for the students. Nevertheless, even with the dice, a lab station can be equipped for a modest cost.`

Procedure

The students cut a 1.25 inch × 5.75 inch (32 × 146 mm) strip of the graph paper. Then, beginning 1 inch (4 squares) from the bottom, they number the lines from 1 to 12 upward through the centre of the strip; see figure 1. Assuming the left edge to be number 1, they mark the next eight vertical lines as 2 through 9, where the right edge is the unlabelled line number 10. The paper clamps are attached to the top and bottom edges of the strip, and the assembly is suspended from a ring stand or other suitable support.

The dice are tossed, and a pair of random numbers are recorded as x (from the 10 sided die) and y (from the 12 sided die) coordinates. A hole is punched at these coordinates as each set of numbers is generated and recorded. (The hole in the bottom of the punch was used to align the punch over the intersection of the co-ordinates. The bale on most punches is easy to remove.) The process is repeated until a punched hole causes the paper to separate by completing the percolation path, a contiguous collection of holes that crossed the strip.

Note: Duplicate coordinates may be recorded in a separate data table. This information may be useful in the discussion of the results. Holes punched at coordinates at the edges of the strip will be half circles. The x-coordinates 1 to 5 must be punched from the left and 6 to 10 must be punched from the right side of the strip. Holes must be punched carefully so as not to disturb the integrity of the rest of the strip. We recommend that the students practise punching holes cleanly before starting the experiment.

Data analysis

Figure 1 shows a representation of a completed strip taken from a typical student data set. To calculate the critical probability for percolation, p_C, the students determine the total number of sites available for punching holes, p, and the number of holes punched in the paper strip to the point where it separated, n, such that

$$p_C = n/p \qquad (1)$$

where $p_C = 68/120$ in figure 1.

The data from all lab groups may be combined and compared with the theoretical value, $p_C = 0.593$ for this case [4, 5]. The level to which this analysis is done will depend on the students and their mathematical abilities.

Final comments

This exercise is accessible to a wide range of students, and all of the participants appear to have fun. A variety of comprehensive questions can be generated, and we enjoy asking the students to extend their interpretations to one and three dimensions, to speculate on the effect of increasing and decreasing the mass of the hanging clamp, and to describe their everyday percolation experiences.

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