

Compensating thermal lensing in Faraday rotators.

Donovan McFeron – University of Florida.
New Physics Building
Corner of Museum and North South Drive
Gainesville, FL 32611

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ABSTRACT

An analyzer cavity was used to test compensation techniques for thermal lensing in the terbium gallium garnet (TGG) crystal in the Faraday isolator for the Laser Interferometer Gravitational-Wave Observatory (LIGO). It was found that placing an FK51 crystal after the Faraday isolator suppressed the mode-mismatch to within 5% of the zero power mode-match values.

INTRODUCTION

Since the mid-19th century, physicists have known accelerating field sources generate waves. An accelerating electric charge generates the familiar electromagnetic waves. Similarly accelerating mass should generate gravitational waves. In 1916, Einstein predicted the existence of gravitational waves. His general theory of relativity, a geometric explanation of gravity, indicated that an accelerating body would generate ripples in the "fabric" of space. These ripples are extremely weak compared to their electromagnetic counterparts. Strong sources of gravity waves are supernovae, colliding black holes, and neutron stars; however, even these sources would move an object on Earth by only 1/10,000 the width of a proton for a fraction of a second. Early attempts to detect gravity waves using resonating bars were unsuccessful, but today physicists use sensitive interferometers, which may detect these waves in the near future.

In Hannover, Germany, German and Scottish physicists run GEO 600, a delay line Michelson interferometer with two 600m long arms. French and Italian physicists are building VIRGO, a Fabry-Perot Michelson Interferometer (FPMI) located in Cascina, Italy. Its arms measure 3km in length. The Japanese also use an FPMI called TAMA 300, a prototype for a future kilometer long interferometer. The length of its arms is 300m, and it is located in Tokyo. Australians run the Australian Consortium for Interferometric Gravitational Astronomy (ACIGA), a prototype FPMI with arms 10m long. All of the above interferometers use a 10W neodymium-yttrium-aluminum-garnet (Nd:YAG) laser running at 1064nm.

The United States operates the largest gravitational wave detectors. NASA and

the European Space Agency (ESA) plan to build the Laser Interferometer Space Antenna (LISA). LISA will act like a giant Michelson interferometer. LISA consists of three spacecraft that form an equilateral triangle with sides 5×10^6 km long. On the ground, the California Institute of Technology (Caltech) and the Massachusetts Institute of Technology (MIT) run the Laser Interferometer Gravitational wave Observatory (LIGO) for the National Science Foundation (NSF). LIGO consists of three interferometers. Two interferometers are in Hanford, Washington, one with 4km arms and the other with 2km arms. The third interferometer is in Livingston, Louisiana, and it has 4km arms. All three interferometers use a 10W Nd:YAG laser at 1064nm. It is planned to upgrade LIGO to increase its sensitivity. This upgrade requires a 180W laser.

The optical components in LIGO are separated into three sections: the Pre-Stabilized Laser (PSL), the Input Optics (IO), and the Core Optics (CO). The PSL runs at 1064nm and 10W. The PSL is designed such that the laser's wavelength and power do not fluctuate. A mode cleaner, which is a ring cavity, provides a nearly pure Gaussian 00 mode beam (see Appendix C).

The PSL sends a beam to the IO for which the University of Florida (UF) takes responsibility. The IO adds sidebands to the laser, which are necessary to detect gravity waves. The IO also isolates the PSL from any back-reflections from the main interferometer. Finally, the IO ensures the size and position of the laser's waist matches those of the Fabry-Perot cavities in the CO.

The CO is the main interferometer. Both arms of the Michelson interferometer (MI) host Fabry-Perot cavities. The cavities are an integer number of times as long as

half the laser's wavelength. The laser resonates inside the cavities unless something changes their lengths. When a gravity wave passes, it causes the cavities' mirrors to move. One arm of the interferometer contracts while the other expands. This imposes a differential phase shift in both arms. Subsequently, the interference pattern in the MI changes.

INPUT OPTICS

The IO links the PSL and CO. Here sidebands are added and the PSL is isolated from back-reflections. Finally, the laser is mode-matched with the cavities in the CO.

Sidebands are used to extract all longitudinal and angular degrees of freedom and to lock the interferometer at its working point. Three Electro-Optic Modulators (EOM's) are used to add sidebands to the laser. In LIGO, the laser runs at 1064nm (2.82×10^{14} Hz), and each EOM adds sidebands at the several MHz range. EOM's rely on optically active crystals. LIGO uses an electro-optic crystal called lithium niobate (LiNbO_3) in its EOM's. When an electric field is applied to LiNbO_3 , its refractive index changes linearly with the electric field, which adds phase modulated sidebands to the field. The electric field is applied transverse to the direction of optical propagation and along a crystal axis[1]. A half-wave plate aligns the polarization of the beam with the crystal axis and electric field. Then the electro-optic effect causes a low frequency electronic signal to be modulated onto the laser beam. Now, the laser has the necessary sidebands for gravitational wave detection.

Isolating the PSL from back-reflections not only prevents the laser in the PSL

from becoming unstable, it also ensures that the subsystems are decoupled. We isolate the PSL using a Faraday isolator, which acts like an "optical" diode. Linearly polarized light can pass through it traveling in the forward direction with no change in polarization. However, that linearly polarized light cannot pass through it traveling in the opposite direction.

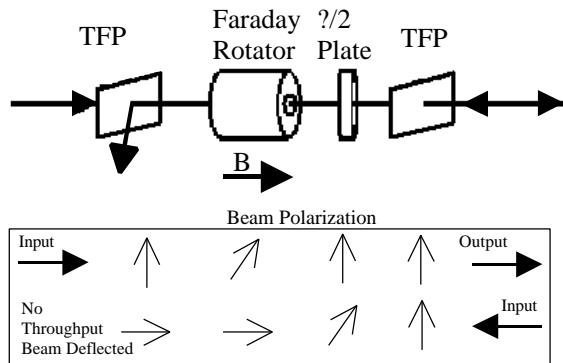


Figure 1. Faraday isolator

A Faraday isolator consists of a thin film polarizer (TFP), a Faraday rotator, a half wave plate, and another TFP (see Fig. 1). The Faraday rotator is the critical component. It rotates the linearly polarized light's polarization by an angle θ . When an isotropic dielectric material is placed in a magnetic field, it becomes optically active[2]. In LIGO, the Faraday rotator uses terbium gallium garnet (TGG). Linearly polarized light propagating through TGG in the direction of the applied magnetic field is rotated by an angle θ . This angle is determined by the equation:

$$\theta = VBl \quad (1)$$

where: l = length of TGG,

B = magnetic field strength in direction of light propagation,

V = Verdet constant for TGG.

This equation stresses that the angle of rotation does not depend on direction of propagation [2].

Finally, the IO mode-matches the laser into the Fabry-Perot cavities (see Appendix A). The modes of the cavities are their eigensolutions, and they depend only on the distance between mirrors and their radius of curvature. The waist and the Rayleigh range are two characteristics of the cavity's mode. If properly mode-matched, when the beam passes through the first mirror, its phase front's match the mirror's radius of curvature, R_1 . Then when the beam reaches the end mirror, the phase front's radius of curvature equals R_2 . The point where the beam's diameter is smallest and the radius of curvature is infinity is called the waist of the cavity [3]. A mode-matching telescope (MMT) matches the size and position of the laser's waist with the cavities'. The MMT consists of two mirrors that change the beam's complex q to match the cavities', which causes the laser to resonate (see Appendix B).

MOVING FROM LIGO I TO LIGO II

Despite similarities between LIGO I and LIGO II, every section of LIGO needs modification. The PSL will be modified to increase power from 10W to 180W, while maintaining a clean beam. Changes in the PSL will require changes in the IO. Increasing the power from 10W to 180W creates a great deal of mode-mismatch. The IO must compensate for this power increase and ensure that the PSL is mode-matched to the CO. Without proper mode-matching the laser will not resonate in the cavities, and LIGO will not detect gravity waves.

Thermal lensing is the main problem that needs correcting. Heating crystals like LiNbO₃ and TGG causes a change in the crystals' index of refraction that depends on the radial distance from the center of the beam and is approximately:

$$n(r) = n_0(1 - \frac{?}{r^2}) \quad (2)$$

where: n_0 = the crystal's initial index of refraction,

$?$ = a coefficient that depends on the crystal,

r = distance from the center of the crystal [4].

The optical path length of an electro-magnetic wave traveling through an object of thickness d with index of refraction n , is $L = nd$. Therefore, the above change in index of refraction leads to a change in optical path length. The optical path length a distance r from the center of the beam is given by:

$$\begin{aligned} L(r) &= nd + \Delta n(r)d \\ &\approx nd + \frac{dn}{dT} \Delta T(r)d \end{aligned} \quad (3)$$

Since the optical path length depends on the distance from the center of the crystal, the crystal becomes a thermal lens (see Fig. 2).

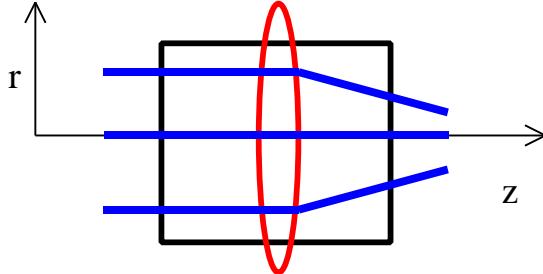


Figure 2. Thermal lensing in a crystal

These additional lenses in the beam line change the complex q of the beam, and mode-mismatch ensues. With a 10W laser thermal lensing is negligible, but with a 180W laser thermal lensing cannot be ignored.

One solution to compensate for thermal lensing in LIGO II is to use a second laser to heat a mirror in the MMT. This MMT directs the beam into the CO. Changing the radius of curvature of that mirror can actively alter the q value of the beam to compensate for thermal lensing. When the PSL's power rises, the power of the laser heating the mirror increases also. The laser heating the mirror must provide a powerful beam with nearly zero fluctuations. Also, the mirror must deform uniformly. Heating a mirror with a second laser is one possible way to compensate for thermal lensing; however this process is complicated.

A simpler way to compensate for thermal lensing requires the introduction of a $-\frac{dn}{dT}$ material. The Faraday isolator is the main source of thermal lensing. We have

not found an isotropic dielectric material to replace TGG. However, we may be able to compensate for TGG's thermal lensing by placing a crystal called FK51 after the Faraday isolator. FK51 has a negative $\frac{dn}{dT}$, while TGG has a positive one. So, when TGG creates a positive lens, FK51 will create a negative one. TGG will act like a bi-convex lens, while FK51 will act like a bi-concave lens (see Fig. 3).

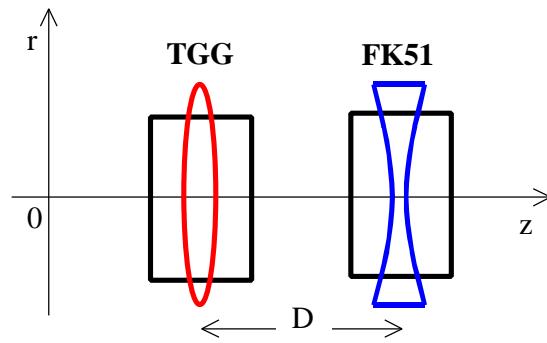


Figure 3. Representation of how FK51 could compensate thermal lensing in TGG.

The total focal length is given by:

$$\frac{1}{f_{tot}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{D}{f_1 f_2} \quad (4)$$

where: f_1 = the focal length of TGG,

f_2 = the focal length of FK51,

If $\frac{D}{f_1 f_2} \approx 0$ and $f_1 = -f_2$, then the net amount of thermal lensing will be zero [5]. This

solution would be independent of power, which would allow LIGO II to run at low power when realigning optical components.

ANALYZER CAVITY TEST SETUP

We use a 1W NPRO (Non-Planar Ring Oscillator) Nd:YAG laser to simulate the PSL in LIGO II. Although the YAG is not as powerful or as clean as the PSL, it is nearly a pure Gaussian 00 beam with some ellipticity. The YAG passes through a Faraday isolator and then a periscope that raises the beam to the height of another laser (see Fig.4). The beam then passes through a half-wave ($\lambda/2$) plate that rotates the beam's polarization and allows it to pass through the TFP. Then a MMT mode-matches the beam into the Faraday rotator that houses the TGG via a horizontal periscope. Before the beam enters the Faraday rotator, it must pass through a TFP that allows only one polarization of light to pass through at the Brewster angle. The YAG passes through a $\lambda/2$ plate and another TFP, which completes the Faraday isolator. A polarizing beam splitter is also in the beam line to help the TFP reject incorrectly polarized light. Then the beam passes through another horizontal periscope and enters another MMT. The MMT mode-matches the YAG into the Fabry-Perot analyzer cavity via the final horizontal periscope. Once the beam enters the cavity it reflects off the mirrors several hundred times. If we properly mode-match the beam with the cavity, then only the 00 mode will resonate and exit the analyzer cavity. However, if the beam has any ellipticity, enters the cavity at an angle, or is not properly mode-matched, or if the cavity mirrors are tilted, then we will see higher order modes. We observe the modes that exit the analyzing cavity in two ways. When the beam exits the cavity it encounters a beam splitter. The beam goes through a focusing lens into a video camera, and the reflected beam goes through an attenuator into a photo diode that is connected to an oscilloscope. We observe

the modes on the camera's monitor and the power levels of the different modes on the oscilloscope.

A 50W Nd:yttrium-lithium-fluoride (Nd:YLF) laser simulates the power output of the PSL. First, the YLF passes through a half-wave plate. Then we use three mirrors to direct the YLF at the first TFP. The TFP reflects the YLF into the Faraday rotator. We adjust the half-wave plate to control how much of the YLF's power the TFP reflects. The YLF heats the TGG and is rejected by the second TFP and polarizing beam splitter. Therefore, the TGG is heated while almost zero YLF light enters the analyzing cavity.

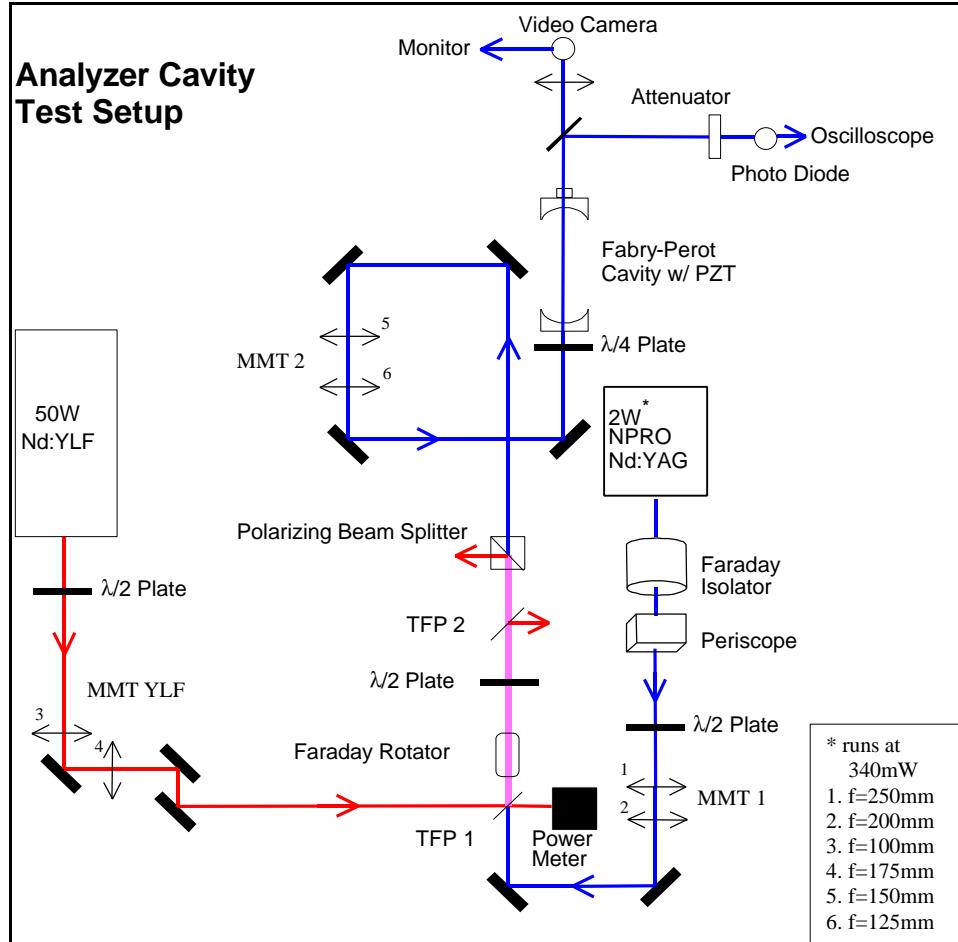


Figure 4. Analyzer cavity test setup.

ANALYZER CAVITY EXPERIMENT

We use the Fabry-Perot cavity to determine how much the q value of the YAG changes. First, without the YLF heating the TGG, we mode-match the YAG with the cavity. We do this by adjusting the MMT, the final periscope mirrors, and the cavity. A Piezo-pusher (PZT) allows the cavity to change its length, thereby cycling through the upper modes. We observe the modes on the monitor and their corresponding amplitude on the oscilloscope. We suppress the “tilt” mode as much as possible. The limit seems to be set by nonsymmetrical contributions in the original laser beam. We measure how well the laser and cavity are mode-matched by measuring the ratio of the “bull’s eye” (BE) to the 00 mode.

After we mode-match the YAG and the cavity with zero YLF power, we adjust the half-wave plate to allow 8W to enter the Faraday rotator. We use the power meter to measure how much power is transmitted through the TFP, and this allows us to determine the amount of power reflected into the Faraday rotator. We allow the TGG to heat for approximately 10 min. Then we align the beam and the cavity to suppress the “tilt” mode by adjusting the final periscope and the cavity. However, we do not adjust the MMT. We then measure the amplitude of the 00 mode and the BE mode. We repeat this process for 16W, 24W, 32W, 40W, and full power.

In LIGO II, FK51 will be placed after the Faraday isolator. However, in our setup there is less than 1W of power after the Faraday isolator. We place the FK51 inside the Faraday isolator between the quarter-wave plate and the second TFP. Our setup allows the YLF to heat both the TGG and the FK51. We repeat the above procedure.

ANALYZER CAVITY RESULTS

First we test the TGG without the FK51. As we increase the power entering the TGG, it drastically changes the characteristic q of the YAG. The amplitude of the 00 mode decreases, while the BE's amplitude increases (See Table I). Thermal lensing in TGG becomes a factor at 16W. The BE rises from a negligible 3.31% of the 00 mode to 33.0% at 42.3%.

TABLE I. Analyzer cavity data using TGG only (10 mode suppressed below 5% of 00 mode).

Reflected YLF power (W)	00 mode amplitude (mV)	BE amplitude (mV)	Ratio of BE to 00 mode
0	667	22.1	3.31%
8.2	664	25.0	3.77%
16.3	639	40.4	6.32%
24.1	601	69.6	11.6%
32.0	534	112	21.0%
40.3	467	142	30.4%
42.3	454	150	33.0%

Next we test the TGG with the FK51. As we increase the power entering the TGG and FK51, the characteristic q of the YAG remains approximately the same. The approximately constant q value is evidenced by the ratio of BE to 00 mode, which does not rise above 4% (See Table II).

TABLE II. Analyzer cavity data using TGG and FK51 (10 mode suppressed below 5% of 00 mode).

Reflected YLF power (W)	00 mode amplitude (mV)	BE amplitude (mV)	Ratio of BE to 00 mode
0	694	16.4	2.36%
8.2	679	9.05	1.33%
16	674	6.95	1.03%
24.1	655	9.40	1.44%
32.3	645	16.5	2.56%
40.4	636	17.5	2.75%
42.1	629	22.0	3.50%

We graph the amplitudes of the BE and 00 modes versus YLF power for both trials (see Fig. 5). We see a sharp drop in the 00 mode and a rise in the BE mode at 16W without FK51. Although the 00 mode drops slightly with FK51, we see that the FK51 compensates most of the thermal lensing in TGG.

We also graph the ratio of the BE mode to the 00 mode versus YLF power (see Fig. 6). Without FK51 compensating TGG, at approximately 16W, the BE mode becomes more than 5% of the 00 mode. However, when we use FK51 to compensate for TGG, the BE mode never rises above 5%.

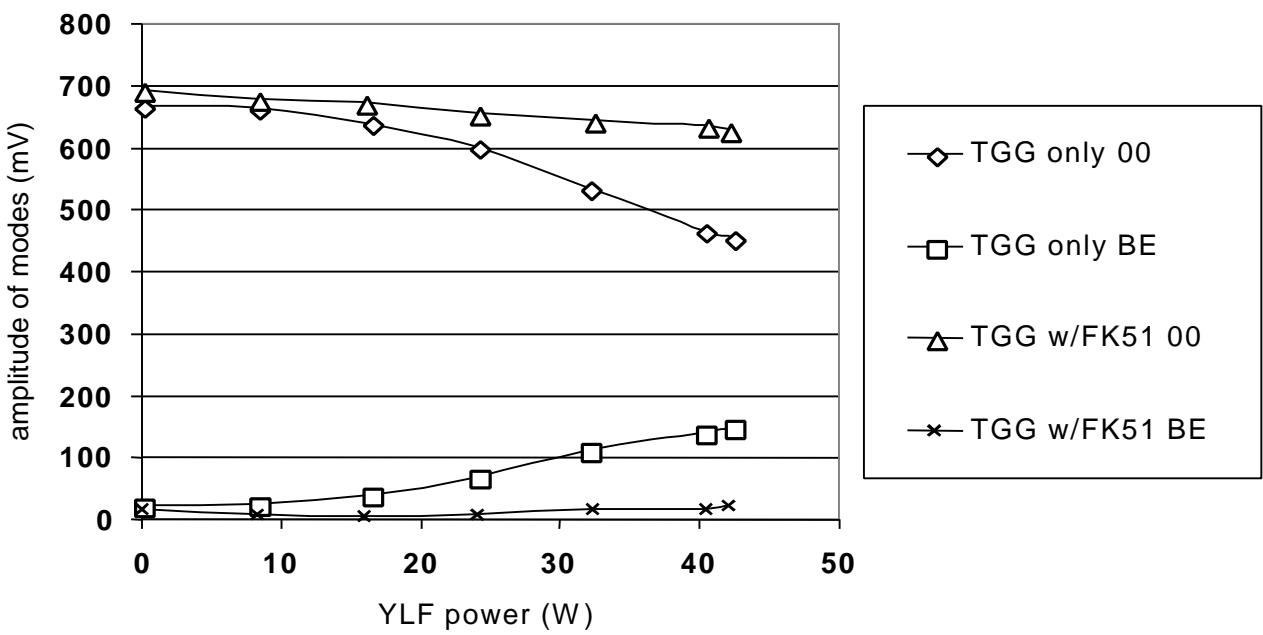


Figure 5. Plot of 00 and BE amplitude versus YLF power both with and without FK51.

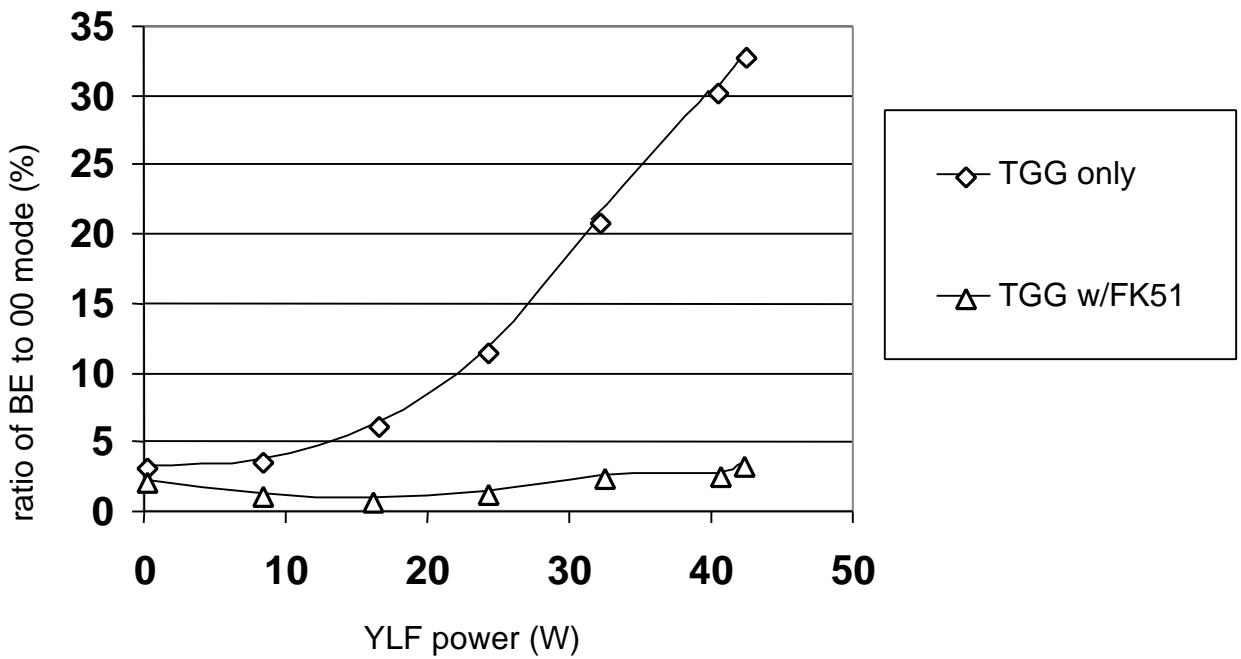


Figure 6. Plot of ratio of BE mode to 00 mode versus power both with and without FK51.

CONCLUSION

From our data we conclude that FK51 compensates for thermal lensing in TGG at high power levels. If we place the FK51 crystal after the Faraday isolator in LIGO II, then we can ensure that the q value of the beam will not change significantly when it goes through the isolator. Further tests should be conducted to determine how well FK51 compensates thermal lensing in TGG at power levels near 180W. We also should determine the relationship between FK51's thickness and its ability to correct thermal lensing. Our experiments show that we can use simple methods to correct thermal lensing in LIGO II.

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APPENDIX A: BEAM / CAVITY CHARACTERISTIC PARAMETERS

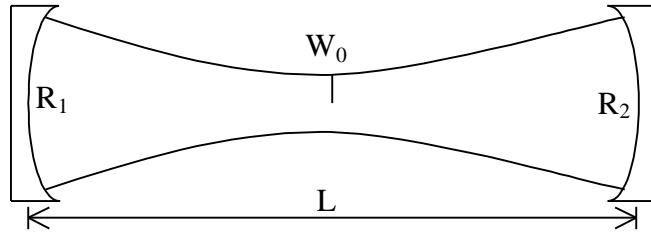


Figure 7. Fabry-Perot cavity.

For a cavity with two mirrors a distance L apart and radius of curvatures R_1 and R_2 , we will define two constants:

$$g_1 \equiv 1 - \frac{L}{R_1} \quad (5)$$

and

$$g_2 \equiv 1 - \frac{L}{R_2} \quad (6)$$

Then the waist of the cavity is:

$$w_0 = \left[\frac{LI}{\mathbf{P}} \sqrt{\frac{g_1 g_2 (1 - g_1 g_2)}{g_1 + g_2 - 2g_1 g_2}} \right]^{1/2} \quad (7)$$

The beam size can be calculated at any point a distance z from the waist by:

$$w(z) = w_0 \left[1 + \left(\frac{Iz}{\mathbf{P}w_0^2} \right)^2 \right]^{1/2} \quad (8)$$

The radius of curvature of the phase front at a point z from the waist is calculated by:

$$R(z) = z \left[1 + \left(\frac{\mathbf{P}w_0^2}{Iz} \right)^2 \right] \quad (9)$$

The propagating Gaussian beam is characterized by a complex q given by:

$$\frac{1}{q_0} = -\frac{iL}{pw_0^2} \quad (10)$$

This is the value of q at the waist. At any point a distance z from the waist q is given by:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - \frac{iL}{pw^2(z)} \quad (11)$$

or

$$q(z) = z - iz_R \quad (12)$$

APPENDIX B: MODE -MATCHING

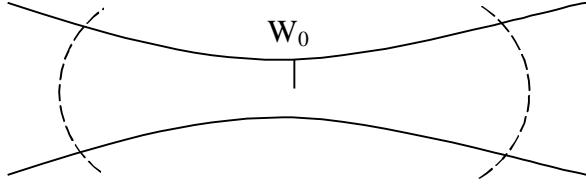


Figure 8. Beam profile.

Mode-matching the beam shown in Fig. 8 with the cavity shown in Fig. 7 is simple to understand. If the beam is mode-matched with the cavity, then its waist will be the same size and in the same position as the cavity's. Also, the radius of curvatures of the beam's phasefront at the position of the cavity's mirrors will be identical with the radius of curvatures of the cavity's mirrors. We mode-match the beam and the cavity by matching their complex q values using lenses or mirrors.

Let q_1 be the beam's complex q, and let q_2 be the beam's new complex q. Then q_2 relates to q_1 by the following formula:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \quad (13)$$

Where A, B, C, and D are given by the following matrices:

For light traveling a distance z in a vacuum:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \quad (14)$$

For light traveling through a curved surface with radius of curvature R and index of refraction n:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{n-1}{R} & 1 \end{bmatrix} \quad (15)$$

For light traveling through a thin lens with focal length f:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \quad (16)$$

APPENDIX C: CAVITY MODES

For a cavity with circular mirrors, the eigensolutions can be described as Hermite-Gauss modes [4]. The fundamental solution for a cavity is the 00 mode. Its intensity is a gaussian distribution (see Figs. 9-10).

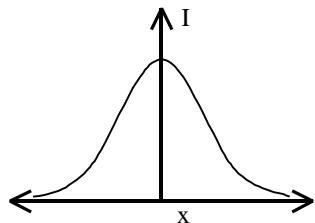


Figure 9. One-dimensional intensity distribution of 00 mode.

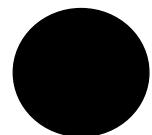


Figure 10. 00 mode as seen by video camera.

The next solution is the 10 Hermite-Gauss mode or “tilt” mode (see Figs. 11-12).

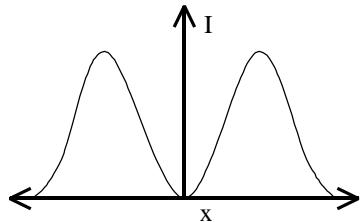


Figure 11. One-dimensional intensity distribution of 10 mode.

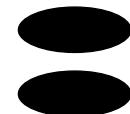


Figure 12. 10 mode as seen by video camera.

The $20 + 02$ Hermite Gauss mode or BE mode is the last solution that we are concerned with (see Figs. 13-14).

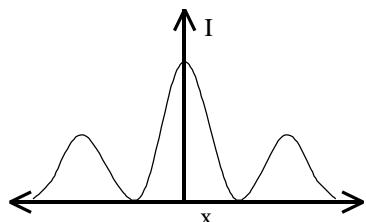


Figure 13. One-dimensional intensity distribution of BE mode.



Figure 14. BE mode as seen by video camera.

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