# Superfluid $^4$He in porous media

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**Abstract**

The progress made in low temperature physics over the past century has been remarkable. In the quest for the lowest temperature two new forms of matter were discovered, superfluids and Bose-Einstein condensates (BEC). In $^4$He, BEC is only *theoretically* believed to be semi-equivalent to superfluidity. Only in a dilute metastable alkaline gas have both BEC and superfluidity been observed, but there have been no quantitative details obtained. By utilizing the effectiveness of a dilution refrigerator and a 25 Å porous silica media the $^4$He gas acts like a diluted gas. The data analyzed, however, did not give a clear relationship, but they did bring about some very interesting questions that may one day lead researchers to the link between superfluidity and BEC.

## 1 Introduction

Dilution refrigeration became important to low temperature physics in the late 1960s. It was a crucial device that allowed for the discovery of $^3$He–$^4$He mixtures phase transition [1]. The refrigerator contains the torsion pendulum which is the primary component in measuring the resonance frequency of liquid helium. The porous media attached to the pendulum affects the resonance frequency in the following way. The normal fluid component of liquid $^4$He will experience drag through the porous media (because the normal fluid component has a non-zero viscosity), hence changing the resonance frequency. Since the superfluid component has zero viscosity then no drag is detected and the frequency is not changed. The significance of this research is to find the link between Bose-Einstein condensates and superfluidity by comparing the change in frequency ($\Delta f$) and the temperature data. The only way that the link between these two phenomena is if the helium behaves like a dilute gas. This condition

is met in the dilution refrigerator, and the torsion pendulum is critical in measuring the resonance frequency.

Initially the plan was to prepare the dilution refrigerator for operation so new frequency and temperature data of the 25 Å porous media could be obtained for analysis. Before the refrigerator could be tested for leaks, a new pump, referred to as 1 K pump from here on, had to be attached to the refrigerator. Once this was completed, Dr. Yasu Takano, his lab assistants, and I began testing the refrigerator for leaks. A large leak was detected inside the vacuum can. Once the leak had been fixed, we were able to evacuate the vacuum can. After the can was evacuated, it was now possible for us to fill the dewar with liquid nitrogen to cool the refrigerator to 77 K. Once the liquid nitrogen cooled the refrigerator, the liquid nitrogen was extracted and the dewar was back-filled with liquid $^4$He. During this process more leaks were detected in the gas handling panel which made obtaining new temperature and frequency data for my project impossible. Therefore, the data that were analyzed for this project were from 1996.

The purpose of this project was to determine, through rigorous data analysis, whether or not the data obtained in 1996 using a 25 Å porous silicon-oxide media followed the same type of trend as the 75 Å data that have been extensively studied by Chan, Reppy, Takano, and others. The non-linear least-square analysis was carried out using Microcal Origin 4.10. This software allowed me to establish the type of equation I was looking to fit, set and vary the necessary parameters, and then graph the line on a plot containing the entire data set.

## 2 Background

- Heike Kamerlingh-Onnes became the first person to ever liquefy helium on July 10, 1908 [1].
- In 1910 to 1911 Kamerlingh-Onnes found the maximum density of helium at the temperature 2.2 K.
- In 1913 Heike Kamerlingh-Onnes was awarded a Nobel Prize.
- In 1924, Bose-Einstein Statistics was generalized from Bose's statistics for an ideal gas for particles having a wave functions that remain symmetrical with respect to the interchange of any pair of particles.
- In 1928, Kamerlingh-Onnes found the allotropic phase transition between He I and He II at $T \simeq 2.2$ K.
- In 1932, Ehrenfest coined the phrase "$\lambda$ point" after a strange anomaly was detected in the specific heat measurements of liquid helium at 2.19 K. The findings were strange because there was no latent heat associated with the $\lambda$ point.
- Independently, P. Kapitza in Moscow and J. Allen and A.D. Misener in Cambridge on January 8, 1938, in the same journal issue, reported immeasurably small viscosity of liquid helium. Kapitza called this *superfluidity*.
- F. London made the suggestion in 1938 that the $\lambda$ transition in liquid He could be identified with the onset of Bose-Einstein Condensation (BEC). Immediately following

this suggestion, L. Tisza proposed the two fluid model in which condensed and uncondensed fractions of the atoms acted as two mutually interpenetrating fluids. Tisza's equations predicted both "second sound" in addition to the ordinary "first sound."

- In 1941, Lev Landau suggested that properties of liquid helium near $T = 0$ could be described in terms of elementary excitations (quasiparticles), phonons (quantized sound waves) and rotons (quantized collective rotational motion of helium).

- In 1944, Peshkov of the Soviet Union provided critical confirmation of the two fluid model. "Second sound" was detected and its propagation velocity was found to be in good agreement with Tizsa'a theory.

- In 1946, the normal fluid fraction of He II was measured.

- In 1948, however, this debate over the liquefaction of $^3$He was settled when researchers at Los Alamos succeeded in liquefying $^3$He, which boiled near 3.2 K at atmospheric pressure.

- The most broadly influential development of the 1950s was the introduction of Landau's Fermi liquid theory, which considered a strongly interacting gas of particles obeying Fermi rather than Bose statistics. This theory provided an accurate model for liquid $^3$He below about 100 mK, provided the foundation for future understanding of superfluidity in $^3$He, and became a powerful general tool for describing many other quantum systems.

- In 1959, "third sound" was produced and observed.

- In 1962, the Nobel Prize in Physics was awarded to Lev Landau for his "pioneering theories for condensed matter, especially liquid helium."

- Also in the 1960's the "fourth sound" was found in porous media where normal fluid was pinned by porous medium. This was a definitive test of superfluidity and established a sensitive method for measuring superfluid density.

# 3 Theory

## 3.1 Superfluidity

Today superfluidity is believed to be a state where bosons actually violate the Pauli Exclusion principle, meaning that the there is more than one particle in an energy state. Superfluid helium was discovered through an attempt to solidify helium gas. At approximately 4.2 K helium gas turns into a liquid (He I) [2]. Researchers continued to lower the temperature to see if liquid helium would then turn into a solid. At 2.17 K liquid helium goes through another phase transition; however, it does not solidify [1]. It turns into a superfluid. This phase of liquid helium is called He II.

The physics of superfluidity depends mainly on thermodynamics. The two most important independent quantities in superfluid $^4$He are the heat capacity, $C$, and the superfluid density, $\rho_s$. The heat capacity of superfluid $^4$He can actually be determined by taking the second partial derivative of the free energy with respect to the temperature (See Appendix A for derivation). The superfluid density is, *theoretically*, the density of $^4$He atoms that are in the ground state. This cannot be directly measured. However, the total density and the normal

fluid density, $\rho_n$, (the density of $^4$He atoms in excited states) can be measured. Although $\rho_s$ cannot be directly measured, there are some facts that are known about it. For example, $\rho_s$ has zero entrophy and zero viscosity, $\eta_s$; $\rho_n$ has non-zero entropy and non-zero viscosity, $\eta_n$. With this information, along with resonance frequency information, it has been determined that the change in the frequency, $\Delta f$, is proportional to the superfluid density within a few parts per million (See Appendix B for a detailed description of this result).

According to the third law of thermodynamics, as the entropy approaches zero, the temperature will approach zero, and vice versa. Then, theoretically, as both the temperature and the entropy go to zero, the ratio $\rho_s/\rho$ will be one. This of course will never happen experimentally because absolute zero cannot be achieved in any laboratory.

The heat capacity can be determined in the following way:

$$C_V = \left( T \frac{\partial S}{\partial T} \right)_V \tag{1}$$

But as the temperature approaches zero the entropy will be a finite value making $\partial S/\partial T$ zero. Hence, the heat capacity will then go to zero. Fig. 1 demonstrates the temperature dependence of the heat capacity. There are two important features to Fig. 1. First, the peak of the graph is at $T_{BE}$; second, as the temperature increases, the graph approaches 3/2; this behavior is similar to that of a classical monatomic gas. Fig. 2 illustrates the condensate fraction, which tells us the number of particles that are in the ground state. The slope of the curve at the y-axis is zero, and the slope of the curve at the T-axis is finite. Fig. 2 is a product of the following equation

$$\frac{N_0}{N_{total}} = 1 - \left[ \frac{T}{T_{BE}} \right]^{\frac{3}{2}} \tag{2}$$

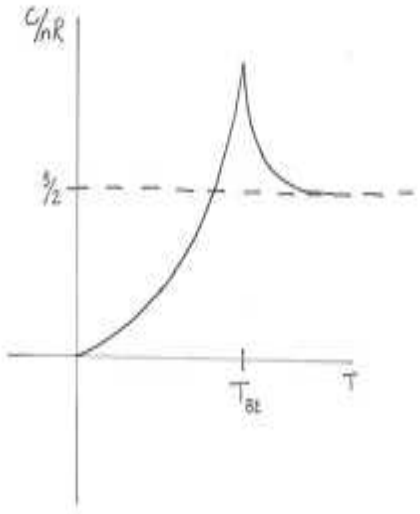


Figure 1: Temperature vs. heat capacity.

There has been a qualitative correlation observed between the number of excited particles and the number of particles in the ground state as well as $\rho_s$ and $\rho_n$. These similarities have provided enough evidence to support the possibility that the $\lambda$ transition temperature of liquid $^4$He is synonymous with the beginning of the Bose-Einstein Condensation [4].

## 3.2 Bose-Einstein Condensates

Bose-Einstein Condensates (BEC) were first predicted by Satyendra Bose and Albert Einstein in the mid-1920's [3]. Einstein took Bose's ideas, which had been initially applied to photons, and applied them to all particles. Einstein's theory only gave new information when it was applied to atoms at very low temperatures [3]. The theory sets out to prove that a large portion of atoms in a gas would fall to their lowest energy state when exposed to very low temperatures. Once the theory was valid, this state at which the majority of the particles will occupy the lowest energy state was called Bose-Einstein Condensates [3].

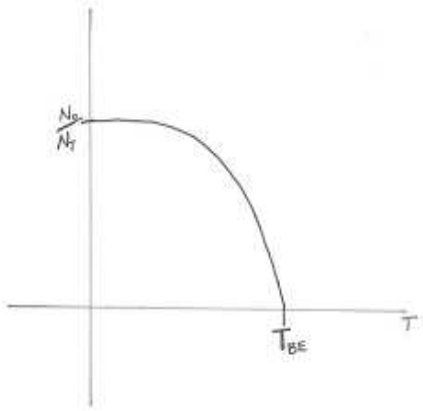


Figure 2: Bose-Einstein condensate graph.

By using Eq. (3) as the main part of the statistics, Bose and Einstein observed that as the temperature of the gas decreased the number of bosons in the ground state increased. This behavior by the atoms is what constitutes a condensate. The equation is as follows:

$$n = \frac{1}{e^{(E_i - \mu/k_B T)} - 1} \tag{3}$$

where $E_i$ is the energy level, $\mu$ is the chemical potential, $k_B$ is Boltzmann's constant, and $T$ is the temperature. Since $\mu$ is less than $E_i$, then the total number of particles is conserved and the following sum rule is obeyed [4]

$$\sum_i \langle n(E_i, T) \rangle = N \tag{4}$$

for all temperatures. When all $N$ particles are in the lowest energy state, $E_0$

$$\langle n(E_0, T) \rangle = \frac{1}{e^{(E_i - \mu/k_B T)} - 1} \simeq N \tag{5}$$

However, when the temperature approaches absolute zero

$$N \simeq \frac{k_B T}{E_0 - \mu} \tag{6}$$

It is known that $N$ is on the order of $10^{20}$; therefore, $\mu$ is just a bit smaller than $E_0$ [4]. Then by moving up just to the first energy level

$$\langle n(E_1, T)\rangle \ll N \tag{7}$$

This relationship implies that $(E_1 - \mu) \gg (E_0 - \mu)$ [4].

After seeing the effects of decreasing the temperature, Bose and Einstein then set out to look for the temperature at which the phase transition actually occurred. This temperature was found to be proportional to the total number of atoms divided by the volume, and that quantity is to a non-integer power

$$T_{BE} = \frac{2\pi\hbar^2}{m_4 k_B}\left[\frac{N}{2.612V}\right]^{\frac{2}{3}} \tag{8}$$

If the particle density that is associated with the saturated vapor of the $^4$He boiling point is put into Eq. (8), then $T \sim 0.5$ K. Since $T \sim 0.5$ K is below the boiling point of $^4$He, then no BE condensates are observed [4]. By using the density of liquid $^4$He in Eq. (8), $T \sim 3$ K [4]. This value is much closer to the $^4$He $\lambda$ transition.

## 4 Dilution Refrigeration

Dilution refrigeration has played a crucial role in the quest for lower and lower temperatures. The principles behind dilution refrigeration are not difficult to understand; however, to construct a dilution refrigerator is not a simple task, but they can be purchased for around $200,000!

The major components in the refrigerator are the 1 K pot, the still, and the mixing chamber

(See Fig. 3). The 1 K pot has a two inch stainless steel pipe that is attached to a vacuum pump that lowers the pressure inside the pot so that the low temperature can be reached in an efficient amount of time. The 1 K pot is where the incoming $^3$He gas in a quarter inch copper pipe is cooled from 4.2 K to 1 K.

A small diameter stainless steel pipe then carries the $^3$He to the mixing chamber where it encounters a pool of liquid $^4$He. The $^3$He diffuses into the $^4$He. The $^3$He-$^4$He mixture is approximately 6% $^3$He [5]. The liquid $^3$He is then "pumped" through a larger diameter stainless steel line to the still where the $^3$He-$^4$He mixture temperature is raised to 0.6 K. From the still the line passes through the 1 K pot and then onto the recovery system.

While the refrigerator contains many other parts, the most important parts are the heat exchangers and the radiation shields. These shields are made of thin sheets of copper or aluminized plastic. The radiation shields between the room temperature and the 4.2 K aid in lowering the temperature inside the dewar to 4.2 K. These gradients also help to keep the liquid $^4$He from evaporating so quickly. By not allowing the $^4$He to evaporate quickly the temperature can be kept fairly low. The radiation shields at temperatures below 4.2 K protect the mixing chamber from the thermal radiation coming from the 4.2 K $^4$He bath.

On top of the dewar is another 2-inch stainless steel line that is used to recover the $^4$He that does evaporate. The line goes from the dewar to the $^4$He recovery system. In addition to the helium recover system there are two mechanisms that are used to control the number of vibrations the dewar is exposed to. One of the mechanisms is a concrete wall that separates the recovery system from the dewar. The blocks are filled with caulking that helps to prevent

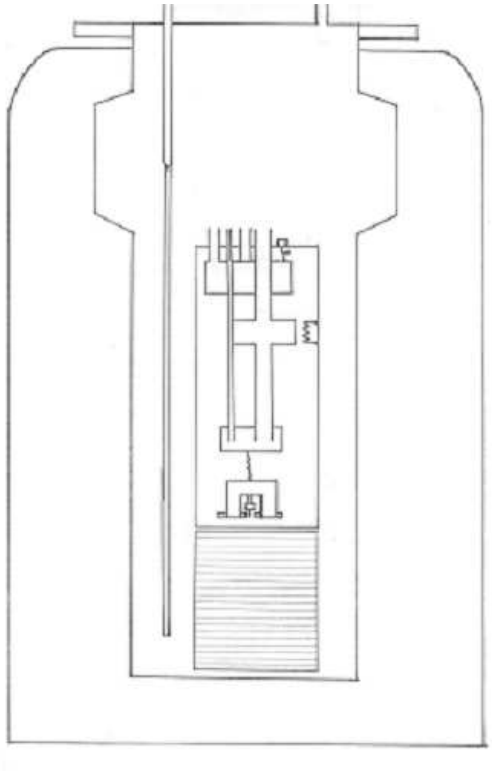

Figure 3: Dilution refrigerator.

the dewar from picking up extra vibrations that could ruin the integrity of the experiment. The three pillars that hold up the dewar also prevent vibrations that are caused from people walking and the noise from the pumps when they are operating.

## 5 Data Analysis

In 1996 there were eight sets of data taken while the dilution refrigerator was running. The cell in this refrigerator contains our sample porous media. The sample is made of silicon oxide ($SiO_2$) and it has 25Å pore diameter. The pores in the media allow the $^4$He to flow through. Each of these data sets will be referred to by the date that it was taken. The sets of data that I concentrated on analyzing were 620 and 707. The data set 620 is believed to

be when the sample cell (i.e. the torsion pendulum bob) was half full of liquid $^4$He. Data set 707 is believed to be when the cell was nearly full of liquid $^4$He.

The reason why I used the 620 and the 707 data sets is explained in Fig. 4. Fig. 4 in Appendix C demonstrates how the superfluid transition temperature and the temperature dependence of the superfluid density evolves as the amount of $^4$He in the porous silica is increased. The 714 data curve is not like the other curves because we believe that there was too much helium present in the cell. This is obvious to us because of the very small zero-point increase of the data and the behavior of the graph near the transition temperature. Between 620 and 707 it seems as if there is something going on, but with the limited amount of data that I had to analyze, it is difficult to say what exactly has happened.

I first had to clean up the data so that all the "bad data points" where sound resonances in the cell affected by the pendulum frequency would not skew the best fit line. Once a fit line could be established, then the "bad points" were put back on the graph and the best fit line was graphed over the top of the data set. By using the software Origin 4.10, I was able to establish the equations and parameters that I wanted to fit the data to. I attempted to see which of two equations the data followed best. The first equation was associated with the behavior of bulk helium:

$$f(x) = P1\left(1 - \frac{x}{P2}\right)^{P3} \tag{9}$$

P1 in this equation is basically the y-intercept of the fit line. P2 is the transition temperature. P3 is the most important of the three parameters. The independent variable in Equation (9), $x$, is the temperature. The independent variable has been determined through research

done earlier by other scientists to have a value very close to 2/3 in bulk helium. The results obtained through the fitting process can be seen in Table I.

TABLE I: Bulk helium parameter values for 620 and 707

| Data Set | P1 | P2 | P3 |
|---|---|---|---|
| 620 | 88.53 Hz ± 0.63 | 349.67 mK ± 0.27 | 0.4595 ± 0.0068 |
| 707 | 119.36 Hz ± 0.45 | 692.4 mK ± 1.4 | 0.6079 ± 0.0069 |

The P3 value for 707 is close to the value for bulk helium, however, the P3 value for 620 is approximately 30% lower than the projected value. Plots of 620 and 707 with this best fit line can be found in Appendix D.

The second equation that we wanted to fit the data to was the Bose-Einstein Condensate equation (See Eq. 2). The equation was basically the same as seen earlier, but there were changes made so that the parameters could be optimized. The equation that was actually fitted to the data follows:

$$f(x) = P1\left(1 - \left(\frac{x}{P2}\right)^{P3}\right) \tag{10}$$

Like the first equation, P1 is synonymous to the y-intercept. P2 in this case is the Bose-Einstein Condensate temperature, and P3 has a value of 3/2 as seen earlier in Equation (2). The 620 data set fit line fit the set beautifully, but the parameter values were not optimal. However, the parameter values that were obtained for the 707 data set were excellent. The P3 value was a little bit high, but the other parameters were very good as seen in Table II. To see plots of 620 and 707 with the BEC fit line, see Appendix E.

TABLE II: BEC parameter values for 620 and 707

| Data Set | P1 | P2 | P3 |
|---|---|---|---|
| 620 | 77.50 Hz ± 0.32 | 364.63 mK ± 0.51 | 3.073 ± 0.032 |
| 707 | 105.85 Hz ± 0.45 | 723.9 mK ± 1.1 | 1.9921 ± 0.0023 |

## 6 Summary

The 620 data set is both BEC-like as well as bulk helium-like. Both expressions describe the data set quite well. The data seem to behave more like bulk helium than BEC when a close look is taken at P3. In the BEC fit line, P3 is approximately 3. This is unphysical because it is twice what the expected BEC P3 value should be. In the bulk helium fit line the P3 value is approximately 30% lower than the projected value; however, it seems more logical that the data are following the bulk helium trend rather than the BEC trend. In the 75 Å diameter pores, the P3 value is very close to the bulk helium value. There most likely is something strange happening between 75 Å and 25 Å.

The 707 data set is BEC-like, but the exponent P3 is very close to 2 instead of 3/2 for non-interacting Bose gas. This behavior leads us to believe that this is a one-dimensional system caused by phonon excitations which determine $\rho_n$.

The phenomena discussed above may possibly be explained as soon as the dilution refrigerator is running and taking acceptable data. Hopefully, this new data, along with this analysis, may shed some light on the relationship between BEC and superfluidity, one of low-temperature physics' oldest puzzles.

## Acknowledgements

I would like to thank Dr. Yasu Takano for teaching me all I ever wanted to know about low-temperature physics. I would also like to thank the National Science Foundation for funding not only the University of Florida's REU but Dr. Takano's research as well. Without the NSF support the learning experience that I had this summer would not have been possible. Last I want to thank Dr. Kevin Ingersent, Dr. Alan Dorsey, and Mrs. Darlene Latimer for all of the work they did to make this research experience possible.

## Appendix A

The Helmholtz free energy of a system must be written in terms of entropy, $S$, and internal energy, $U$.

$$F \equiv U - TS \tag{A.1}$$

The heat capacity, $C$, by definition is

$$C \equiv \lim_{\Delta T \to \infty} \frac{\Delta Q}{\Delta T} \tag{A.2}$$

If the volume is held constant, the heat capacity is written

$$C_V = T \left( \frac{\partial S}{\partial T} \right)_V \tag{A.3}$$

From the fundamental definition of entropy, temperature and conservation of energy the following is derived

$$dU = TdS - PdV \tag{A.4}$$

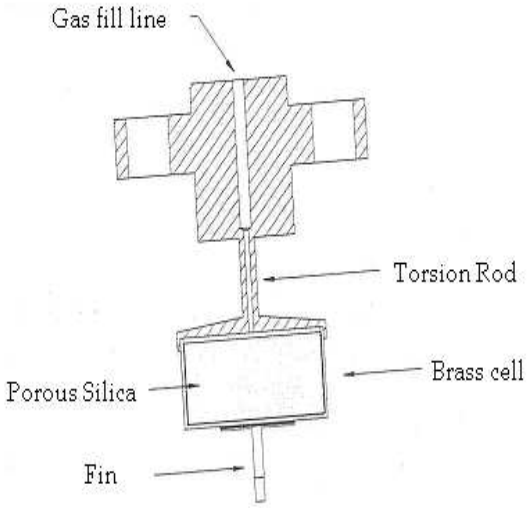


Figure 4: Torsion pendulum used in this experiment [6].

Combining Equations (A.1) and (A.4)

$$dF = -SdT - PdV \tag{A.5}$$

Since the volume is being held constant, the entropy can be written

$$S = -\left(\frac{\partial F}{\partial T}\right)_V \tag{A.6}$$

Then by substituting Equation (A.6) into Equation (A.3) we find that

$$C_V = -T\left(\frac{\partial^2 F}{\partial T^2}\right)_V \tag{A.7}$$

## Appendix B

The torsion pendulum that was used in this experiment is shown in Fig. 4.

The resonance frequency of the system is

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{B.1}$$

where $k$ is the spring constant and $m$ is the mass. However, we know from mechanics that translational entities, such as mass, are synonymous to rotational entities, such as moment of inertia. The resonance frequency can be derived after a few lines of simple calculus to give

$$\omega_0 = \sqrt{\frac{k_t}{I}} \tag{B.2}$$

where $k_t$ is the spring constant due to the torsion pendulum. The total moment of inertia, like the total density, has several parts.

$$I = I_B + I_n + I_s \tag{B.3}$$

$I_B$ is the moment of inertia of the sample cell and porous media; $I_s$ is the moment of inertia of the superfluid component of $^4$He; and $I_n$ is the moment of inertia of the normal component. $I_s$ does not participate in the torsional motion of the pendulum because it has zero viscosity, $\eta_S$=0, and will not be dragged along with the pendulum; therefore, only $I_B$ and $I_n$ enter into Equation (B.2) which can be re-written as

$$I = I_0 - I_s \tag{B.4}$$

where $I_0$ is the total moment of inertia. Now the resonance frequency can be written

$$\omega_0 = \sqrt{\frac{k_t}{I_0 - I_s}} \tag{B.5}$$

Factoring out $\sqrt{\frac{k_t}{I_0}}$ then allows us to determine the first order expansion of the right hand side of Equation (B.5)

$$\omega_0 \simeq \sqrt{\frac{k_t}{I_0}} \left[1 + \left(\frac{1}{2}\right)\left(\frac{I_s}{I_0}\right)\right] \tag{B.6}$$

The frequency "shift" relative to the frequency above the superfluid transition

$$\Delta f = \frac{1}{2\pi}(\omega - \omega_0) \tag{B.7}$$

where $\omega_0$ is the angular resonance frequency of the pendulum when the helium is in the normal, non-superfluid state, and $\omega$ is the angular resonance frequency when the helium is in the superfluid state. Finally, the change in frequency can be written as a function of the resonance frequency and the moment of inertia

$$\Delta f = \frac{1}{4\pi}\frac{\omega_0}{I_0} I_s \tag{B.8}$$

Since $I_s$ is proportional to $\rho_s$ then it is known that the change in frequency is proportional to the superfluid density; therefore

$$\rho_s \propto \Delta f \tag{B.9}$$

implying that the frequency change is the superfluid density!

# Appendix C

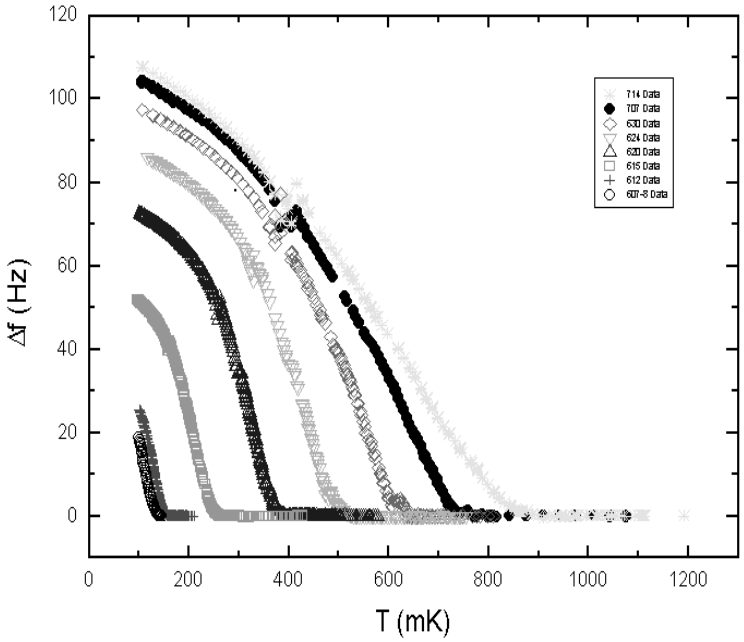


Figure 5: Data taken for different amounts of $^4$He in the porous silica.

# Appendix D

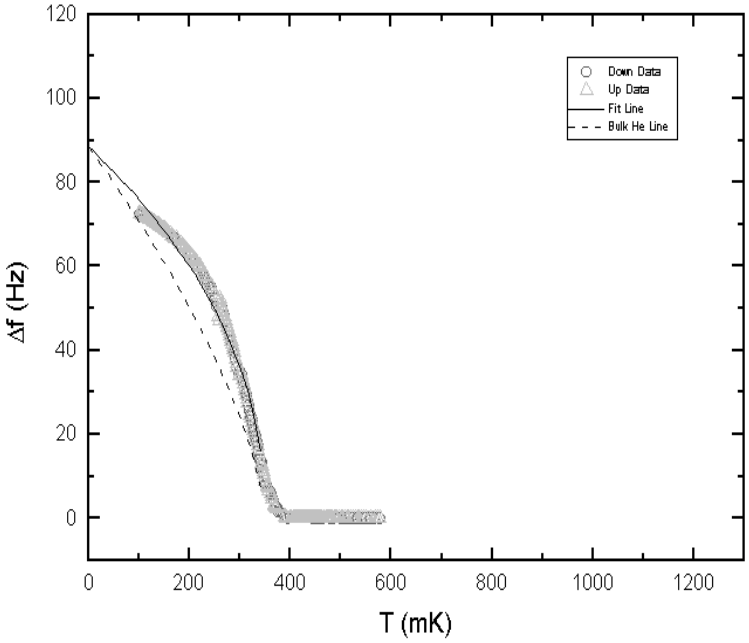


Figure 6: 620 data set plot with bulk helium fit line.

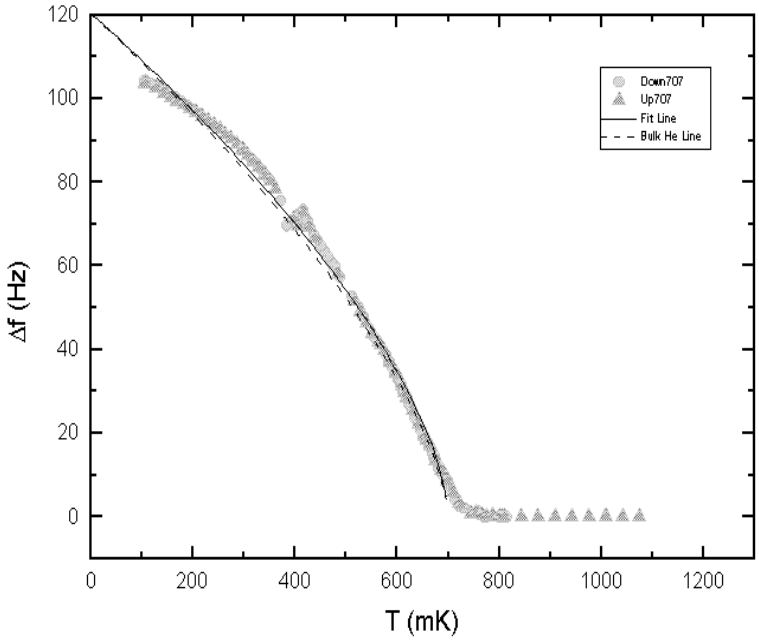


Figure 7: 707 data set plot with bulk helium fit line.

## Appendix E

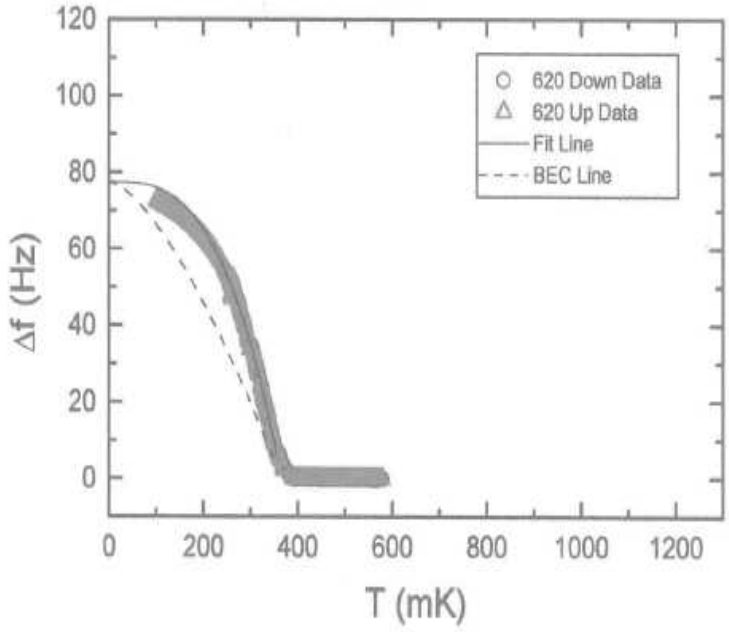


Figure 8: 620 data set plot with BEC fit line.

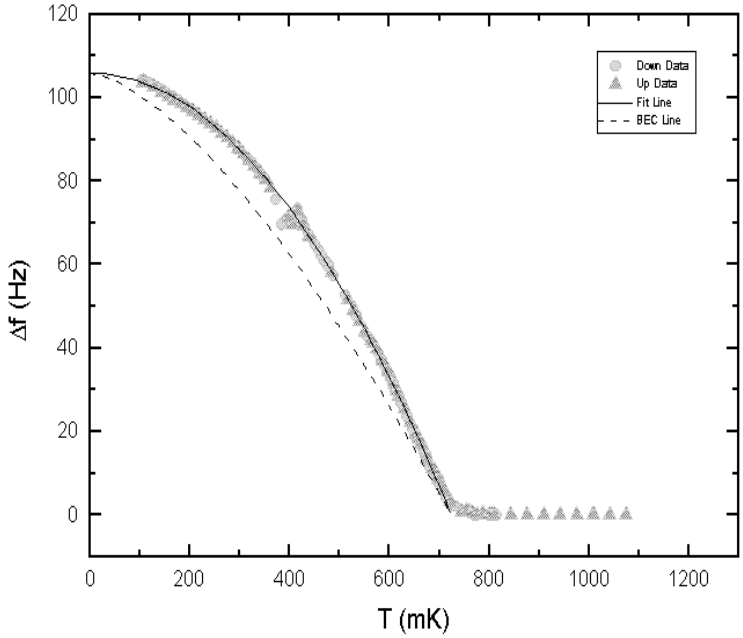


Figure 9: 707 data set plot with BEC fit line.

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