Characterization of a tunable cavity for the LISA mission

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Abstract

The Laser Interferometer Space Antenna (LISA) is a gravitational wave detector aiming to detect low frequency gravitational waves. To do this, LISA will need to overcome a large degree of laser phase noise. Stabilizing a 1064-nm Nd:Yag laser to a tunable high-Finesse optical cavity could eliminate much of this noise. This project investigated the feasibility of using a tunable piezoelectric (PZT) cavity and its application to the LISA mission by comparing the stability of a PZT-actuated cavity with the stability of non-actuated cavities. This paper characterizes and presents first results of a tunable cavity that could be used for the first step of pre-stabilization.

Introduction

The Laser Interferometer Space Antenna (LISA), a joint NASA/ESA project, is a space-based gravitational wave observatory that will detect gravitational waves from merging super-massive black holes and neutron star binaries. The primary objective of LISA will be detecting low-frequency gravitational waves in the 30-μHz to 1 Hz frequency range [1]. LISA will consist of three spacecraft forming a triangle with 5-Gm arm lengths and will move in heliocentric orbit 20 degrees behind Earth's orbit (*See Figure 1*). Each spacecraft will contain two freely falling test masses shielded from outside disturbances. Gravitational waves passing through LISA will lengthen one arm relative to another. The change caused by a typically expected gravitational wave in the LISA frequency band has a strain of about 10^{-21} m/Hz$^{1/2}$ [1]. When passing through LISA this change is equal to the change in length over the arm length, $\frac{\Delta L}{L}$, or about $\frac{5^{-12}m}{5Gm}$ [1]. LISA will measure the phase changes in the interferometer signal between test masses in adjacent spacecraft due to these changes in arm length.

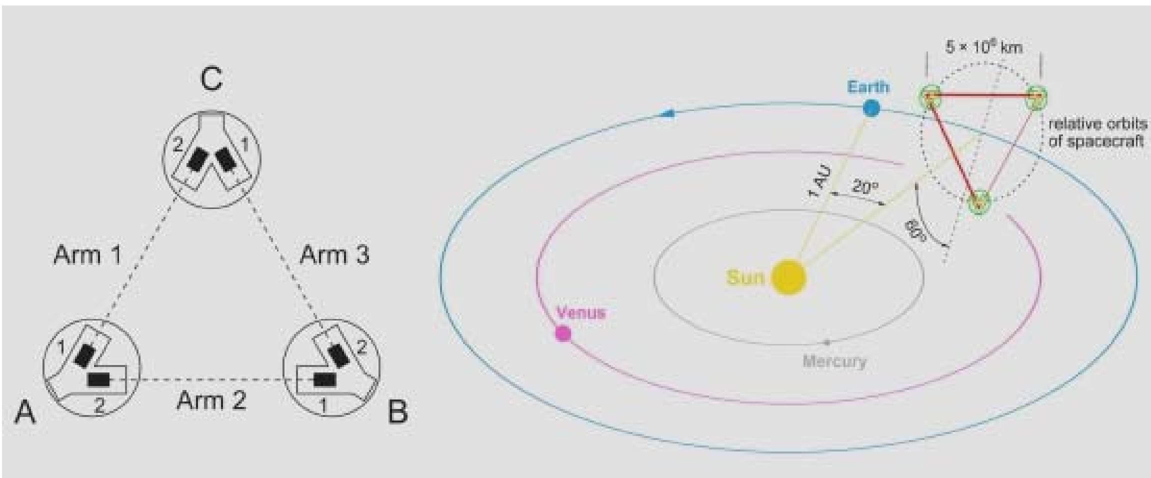

Figure 1: Representation of three spacecrafts and LISA heliocentric orbit[2].

One challenge that faces LISA interferometry is that the laser phase noise is several orders of magnitude above an expected signal from a gravitational wave. The interferometer signal depends on the phase of the received laser field:

$$\rho = kL = \frac{\omega L}{c} = \frac{(\omega_0 + d\omega)(L + dL)}{c} = \frac{\omega_0 L}{c} + \frac{d\omega L}{c} + \frac{\omega_0 dL}{c} \tag{1}$$

where $d\omega$ is the laser frequency noise and dL is the gravitational wave induced length change. Therefore $d\omega L < \omega_0 dL$. Since the relative noise is equal to the relative length change in LISA, we can derive the requirement for the frequency noise if we would perform a single arm measurement.

$$\frac{\Delta \upsilon}{\upsilon} = \frac{\Delta L}{L} \tag{2}$$

$$d\upsilon = \frac{\upsilon dL}{L} = \left(3 \times 10^{14}\, Hz\right)\left(10^{-21} / \sqrt{Hz}\right) = 3 \times 10^{-7}\, Hz / \sqrt{Hz} \tag{3}$$

Using two arms in a Michelson-type interferometer, we can then subtract the phase from two arms to find our desired laser stability for LISA.

$$\rho_1 - \rho_2 = k\Delta L \tag{4}$$

$$k\Delta L \rightarrow d\upsilon < \frac{\upsilon_0 dL}{\Delta L} = \frac{\left(3 \times 10^{14}\, Hz\right)\left(5 \times 10^{-12}\, m / \sqrt{Hz}\right)}{5 \times 10^{7}\, m} = 3 \times 10^{-5}\, Hz / \sqrt{Hz} \tag{5}$$

The desired laser stability of $3\text{x}10^{-5}$ Hz/Hz$^{1/2}$ can be attained in three steps of stabilization [2]. The first step to reduce laser frequency noise is pre-stabilization. Pre-stabilization provides a frequency reference for the LISA lasers. To provide this reference, the laser frequency must be stabilized to an ultra-stable optical cavity formed between two highly reflective mirrors. In this pre-stabilization, a stability of 30 Hz/Hz$^{1/2}$ or better above 3 mHz should be achieved [2].

The next step in noise suppression of the laser is arm-locking, accomplished by stabilizing the laser frequency to one or more of LISA's arms. Arm-locking is the most stable reference in our frequency band.

The residual frequency noise can then be further reduced using a technique known as time-delay interferometry. In LISA, each spacecraft sends and receives light to and from each of the adjacent spacecraft. The received light is combined at a photodetector with light from a reference laser with the frequency difference recorded as beat signals. The beat signals from each of the three spacecraft are delayed in time and recombined, essentially creating three virtual equal-arm Michelson interferometers canceling the remaining three to four orders of magnitude of laser frequency noise [3].

As the LISA constellation orbits the sun, a periodic Doppler shift in the frequency will be induced due to the length contractions/expansions in the LISA arms. The center of mass of LISA will orbit the Sun with a period of one year. As the bulk motion orbits the Sun, the triangular formation will rotate about its center in a clockwise manner. Each spacecraft orbit, slightly elliptical and slightly tilted with respect to the other individual orbits and to the plane of the Earth's orbit, will maintain the triangle formation of LISA (*See Figure 2*) [4]. The changes in the Doppler shift caused from this yearly variation demand tunable pre-stabilization to combine it with arm-locking.

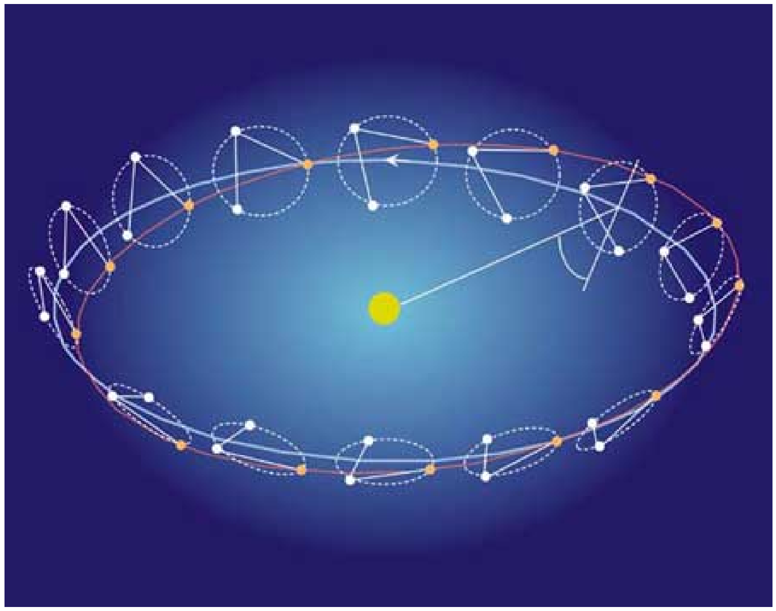

Figure 2: LISA's orbit over a full year[4].

To achieve the required high-stability and tunability of the cavity, we test the cavity's stability with a piezoelectric crystal (PZT) that allows control over the length of the cavity. A voltage applied to the PZT is proportional to a change in the PZT length. A fluctuating voltage would cause the cavity length to fluctuate resulting in frequency noise. Our aim is that the additional noise from the voltage should not increase the frequency of the cavity significantly beyond the 30 $Hz/Hz^{1/2}$ requirement.

Experimental Setup

Since the estimated temperature stability on the LISA optical bench will be a few $\mu K/Hz^{1/2}$ at 1 mHz, we must achieve a similar environment for the cavity in our vacuum tank. To do this, we placed the cavity inside a vacuum tank with five layers of gold-coated stainless steel to provide the necessary insulation. A 20-cm reference cavity made of a

Zerodur spacer was assembled with a flat mirror and a mirror with radius of curvature of 0.5 m, both with high reflectivity and with an anti-reflection (AR) coating, optically contacted to the ends. The laser locked to this cavity was used as a frequency reference to compare other materials' stability to. This step had already been done by previous group members and was used as a reference for our tunable cavity (*See Figure 3*).

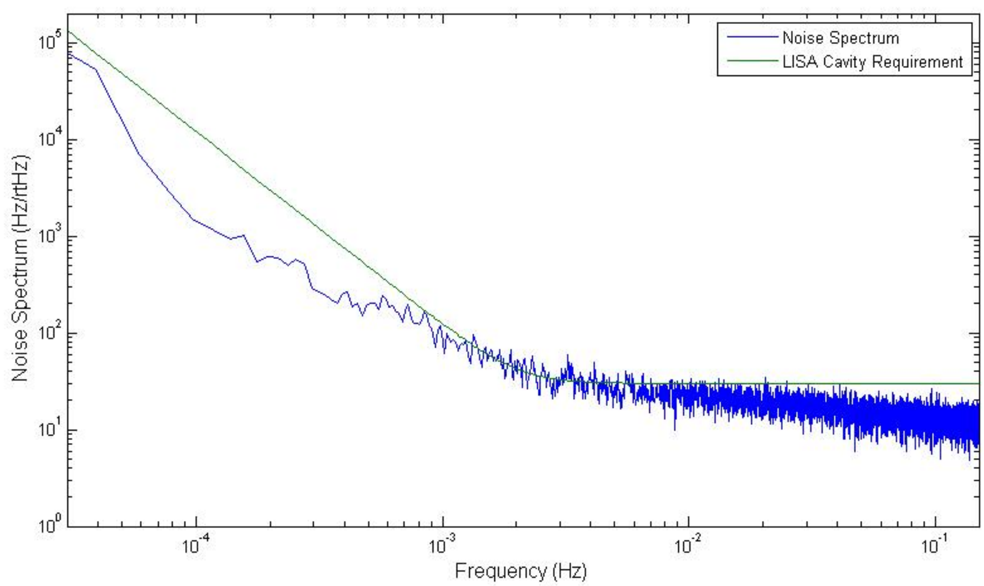

Figure 3: Results from prior experiment show the noise spectrum of the reference cavity can be stabilized to meet the LISA cavity requirement.

To test our cavity, we stabilized a 1064-nm Nd:YAG laser was using the Pound-Drever-Hall (PDH) technique, discussed later in this paper. We made the cavity using a flat mirror and a 0.32-cm PZT hydroxide-bonded to one end of a 5.0-cm Zerodur spacer, and a mirror with a radius of curvature of 1 m optically contacted to the other end of the spacer (*See Figure 4*). The free spectral range (FSR) of our cavity, c/2L, was then found to be 2.78 GHz.

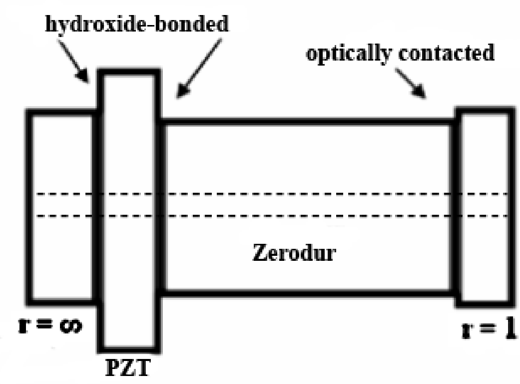

Figure 4: PZT cavity construction, consisting of a flat mirror hydroxide bonded to piezoelectric crystal (PZT), PZT hydroxide bonded to Zerodur spacer, and a curved mirror optically contacted to spacer.

We achieved a visibility of 79% showing that the spatial mode of the laser is well matched to the spatial eigenmode of the cavity. Characterizing our cavity we found it to have a linewidth of 0.4 MHz (FWHM: full width half max), and a Finesse, $\frac{FSR}{FWHM}$, of 6,900. The following equations show how the Finesse we found compares with the expected Finesse. This expected finesse is calculated:

$$F = \frac{\pi\sqrt{grt}}{1-grt} \tag{6}$$

where grt is the magnitude of our round trip gain.

$$grt = r_1 r_2 \tag{7}$$

r_1 and r_2 are the amplitude reflectivities of the cavity mirrors. In our case our mirrors have symmetric coating and $r_1 = r_2$, thus $r^2 = R$ and $1 - R = T$, the intensity transmission of each of the mirrors. With high reflectivity, such as in our case, r = 1.

$$F = \frac{\pi r}{1 - r^2} = \frac{\pi r}{T} = \frac{\pi}{T}$$

(8)

Our expected T is about 250 ppm, giving us an expected Finesse of 12,500. With our calculated Finesse of 6,900, we are getting a T of approximately 455 ppm, which is pretty good. We then found our PZT coefficient to be 2 MHz/V which demands our voltage noise to be lower than 15 μV/ $Hz^{1/2}$ to meet the 30-Hz/Hz$^{1/}$2 requirement.

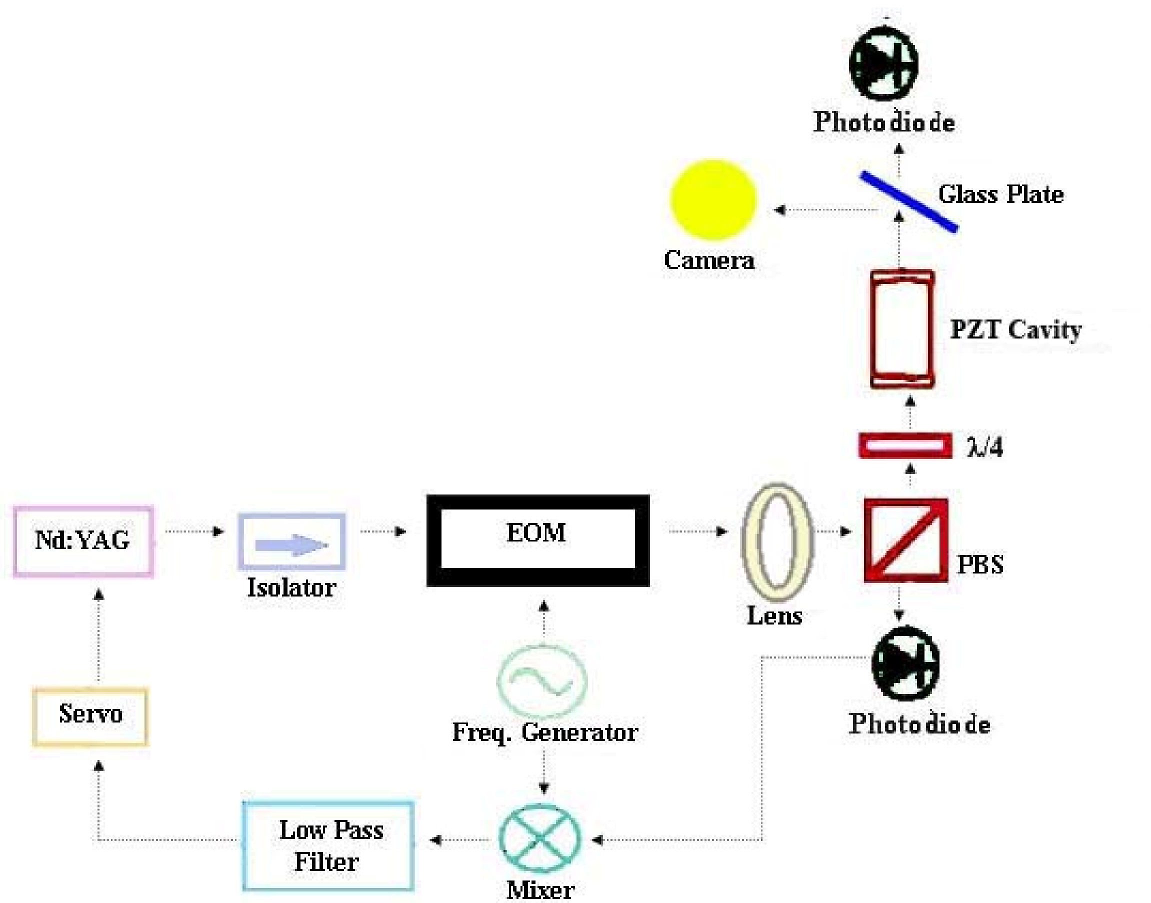

Figure 5: Pound-Drever-Hall layout, including an electro-optic modulator (EOM), a polarizing beam splitter (PBS), and a quarter wave plate (λ/4).

The PDH layout is shown schematically in Figure 5. To stabilize our laser to the cavity, the laser beam passed through a Faraday isolator which prevents internal distortions from other optical components reflecting light back into the laser. The beam then traveled through a lens used to mode-match the spatial profile of the laser to the eigenmode of the cavity. The lowest-order transverse mode is desired because it will allow for the lowest diffraction losses. To mode-match the profile of the laser to that of the cavity we first found the waist size of the cavity (ω_c). The length (L) of the cavity is 5.4 cm and contains two mirrors; one with a 1m (R_b) radius of curvature and a flat mirror (R_f). The wavelength of the laser in free space (λ) is 1064 nm. Before finding the waist size of the cavity we had to define the g parameters, g_1 and g_2, given by:

$$g_1 = 1 - \frac{L}{R_f} \text{ and } g_2 = 1 - \frac{L}{R_b} \tag{9}$$

The waist of the cavity was then calculated with the following formula:

$$\omega_c = \left(L \frac{\lambda}{\pi} \sqrt{\frac{g_1 g_2 (1 - g_1 g_2)}{(g_1 + g_2 - 2 g_1 g_2)^2}} \right)^{1/2} \tag{10}$$

and found to be 276.55-μm.

We next needed to find the size and locations of the laser's waist by profiling the beam out of the fiber. This was done by focus the beam onto a photodiode and placing a chopper at various places along the beam's path giving us a plateau graph of voltage vs. time on the oscilloscope. After calculating the voltage at 84% and 16% to get the time distance (τ) between where they met on the graph, this distance is directly related to the waist size, the data was analyzed using the formula:

$$\omega_0 = \frac{2 f \pi r t}{\text{\# of slots of chopper}} \tag{11}$$

where f is the frequency of the chopper and r is the distance from the center of the chopper to where the beam passes through it. The waist size (ω_0) of the beam was found to be 227.87-μm.

With the beam size of the laser known, we then had to match it to the cavity using a lens with a specific focal point at a specific location along the beam's path. We planned for the cavity's waist, which would be at the flat mirror of the cavity, to be about 1.944 m from our source. Using a file we created in Mathematica, we matched it using one lens with the equation for expansion of the beam:

$$\omega_{(z)} = \omega_0 \left(1 + \left(\frac{\lambda z}{\pi \omega_0^{\ 2}} \right)^2 \right)^{1/2} \tag{12}$$

The waist of our beam, ω_0, is a distance z_0 away from our flat mirror and the waist of the cavity, ω_c, is directly in front of our other mirror. A distance z_1 is measured from ω_0 to the center distance between the two mirrors. A second distance, z_2, is measured from the center to ω_c, or the distance between ω_0 and ω_c minus z_1. We used the above equation for z_1 and z_2 replacing ω_0 with ω_c for z_2. We then set the two equations equal to solve for z_1 and z_2 and were able to calculate the radius of curvature (R_1 and R_2) pertaining to each distance.

$$R_1 = z_1\left(1+\left(\frac{\pi\omega_0{}^2}{\lambda z_1}\right)\right) \tag{13}$$

and

$$R_2 = z_2\left(1+\left(\frac{\pi\omega_c{}^2}{\lambda z_2}\right)\right) \tag{14}$$

These results were then used to calculate for the focal length (f),

$$\frac{-1}{R_2} = \frac{1}{R_1} - \frac{1}{f} \tag{15}$$

which we found to be .499 m. A close lens to this focal point was .50 m. By plugging this in for the focal length we calculated a distance of .88 m to place the lens from the waist of our laser so that our mode would match that of the cavity. We set up our lens on a slider to make gradual adjustments of mode matching into the cavity and were able to profile to beam to have waist size of 305.31-µm at distance -.758447 m from our source (*See Figure 6*). Since our source was located past our cavity, we got a negative distance. Further adjustments in alignment were made later.

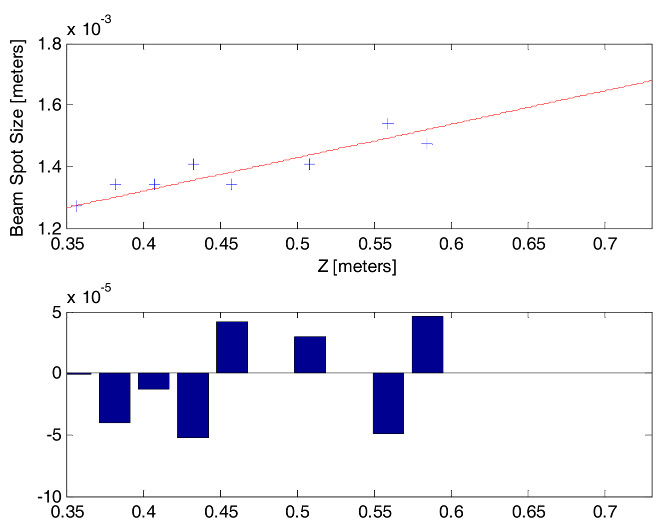

Figure 6. Calculated beam size and location of the laser's waist with lens.

The laser next passes through an electro-optic modulator (EOM) to modulate the phase of the carrier frequency which produces sidebands of 24.3 MHz. We can see in the equation for the electric field of the laser how these sidebands are produced. Equations 16 and 17 show the the electric field of the beam before and after it passes through the EOM.

$$E_L = E_0 e^{i\omega t} \tag{16}$$

$$E_{EOM} = E_L e^{im\sin(\Omega t)} \tag{17}$$

We can use a Taylor expansion to linerize this equation.

$$E_{EOM} = E_L\left(1 + im\sin(\Omega t) + \ldots\right) = E_0 e^{i\omega t}\left(1 + im\sin(\Omega t)\right) \tag{18}$$

where,

$$\sin(\Omega t) = \frac{1}{2i}\left(e^{i\Omega t} - e^{-i\Omega t}\right) \tag{19}$$

$$E_{EOM} = E_0 e^{i\omega t}\left(1 + \frac{m}{2}\left(e^{i\Omega t} - e^{-i\Omega t}\right)\right) = E_0 e^{i\omega t} + E_0\frac{m}{2}e^{i(\omega+\Omega)} - E_0\frac{m}{2}e^{i(\omega-\Omega)} \tag{20}$$

After passing through the EOM, the laser light then traveled through a polarizing beam splitter and quarter-wave plate acting like an optical diode. The quarter-wave plate created a quarter-wavelength phase shift between itself and the cavity which changes linearly polarized light to circularly polarized light, and vice versa. The net effect of the polarizing beam splitter and quarter-wave plate was to separate the back-reflected light from the transmitted light which was used to lock the cavity. The light transmitted through our cavity hit a glass plate sending the majority of the light onto a photodiode.

We used this light for alignment purposes. The rest of the transmitted light went to a camera used to view the spatial mode of the cavity's internal field. The light reflected from the cavity passed back through the quarter-wave plate and polarizing beam splitter and was detected by a photodiode with a bandwidth of 1.5 GHz.

The signal from a frequency generator running at 24.3 MHz was sent to both the EOM and a mixer which also had an RF input from the fast photodiode. The output of the mixer passed through a low-pass filter to filter out the higher harmonics of the generated 24.3-MHz signal and provided us with our error signal. When the laser frequency was on a cavity resonance, the laser carrier frequency would match to the cavity resonance and the corresponding error signal was zero volts. As the two sidebands pass through the fast photodiode, they produced two heterodyne beats that were 180˚ out of phase with each other when the laser frequency was on resonance and cancel out. Because the phase modulator swept the phase of the laser carrier at a frequency much higher than the linewidth of the cavity, in our case 0.4-MHz, the sidebands were directly reflected with no phase shift. When the laser frequency was not on resonance, the two beats from the sidebands were no longer 180˚ out of phase with each other and our error signal read as some positive or negative voltage (*See Figure 8*). We used this non-zero voltage to correct the laser frequency such that it matched the resonant cavity frequency. We then sent the error signal to the actuator where the beam was locked.

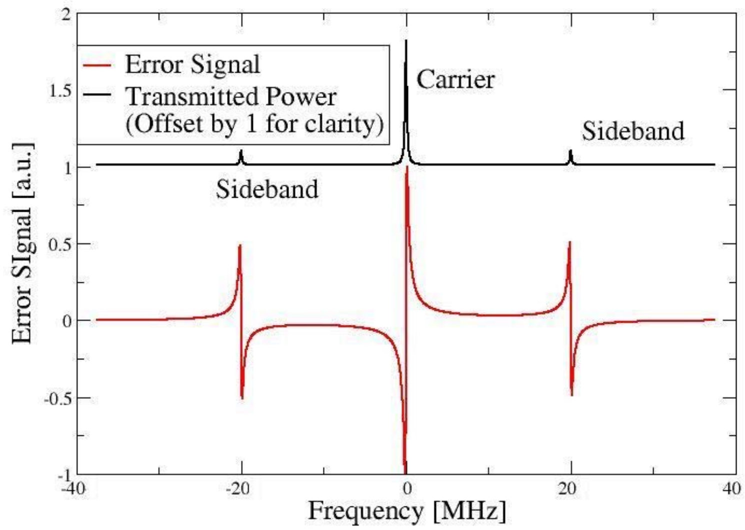

Figure 8: Error signal as phase modulator swept over the phase of the laser carrier and the sidebands.

By using this error signal we kept the laser frequency stabilized to the resonant frequency of the cavity. After stabilizing the laser to our cavity we compared it with another laser that had been stabilized to the reference cavity using a PDH technique. We aligned pickoffs from both lasers on top of each other. The two laser fields beat against each other and were detected by a photodiode where the resultant frequency was measured by a frequency counter and recorded with a computer (*See Figure 9*). By doing this, we tracked the relative stability of our two cavities and compared them to the requirements for the LISA mission.

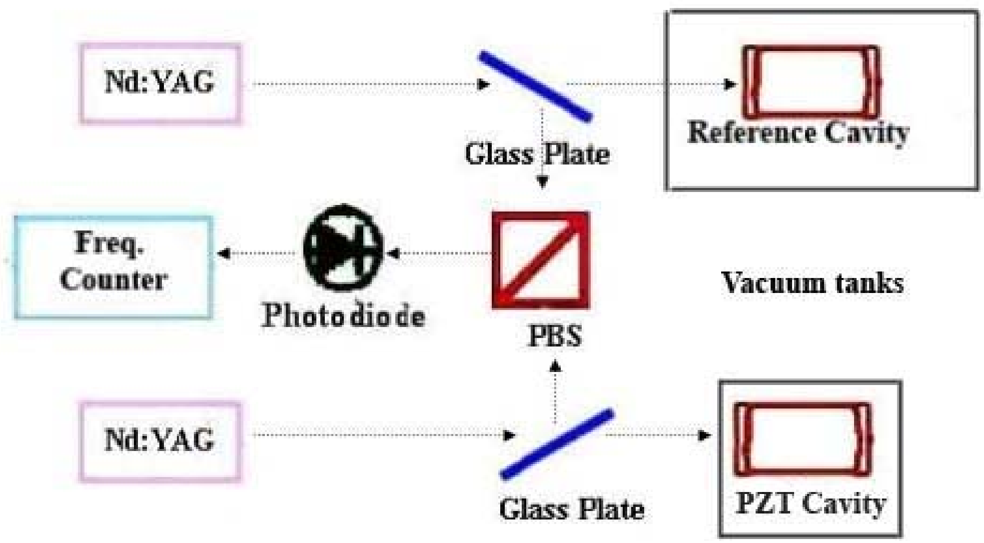

Figure 9: Two cavities beating against each other.

Results

The time series of the beat-frequency drifts (*See Figure* 10), most likely due to temperature changes within the tank caused by the changing ambient temperature. The PZT material of the cavity is only 1/16 the length of the Zerodur material which causes the PZT cavity to be dominated by the coefficient of thermal expansion (CTE) of the PZT material, 4 x 10^{-6} m/K [5]. Since the PZT CTE is larger than the CTE of the Zerodur, 2 x 10-8 m/K [6], a change in temperature will therefore cause the PZT cavity to change length more than the reference cavity, and thus the frequency of the PZT cavity will also change more. Figure 11 shows the noise spectrum of the two cavities without a voltage applied to the PZT. The green line is the LISA cavity requirement.

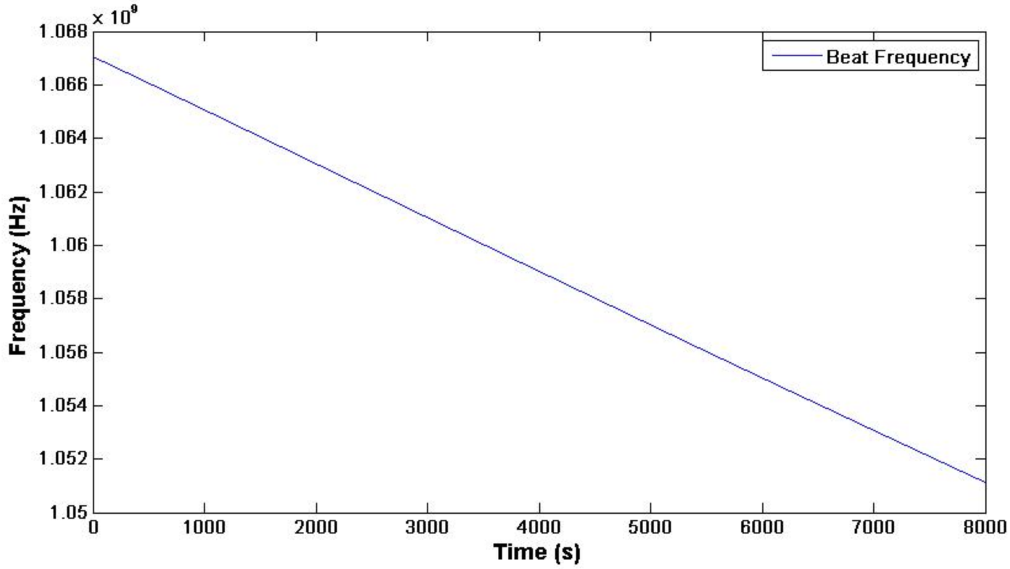

Figure 10: Time series of beat frequency showing drift.

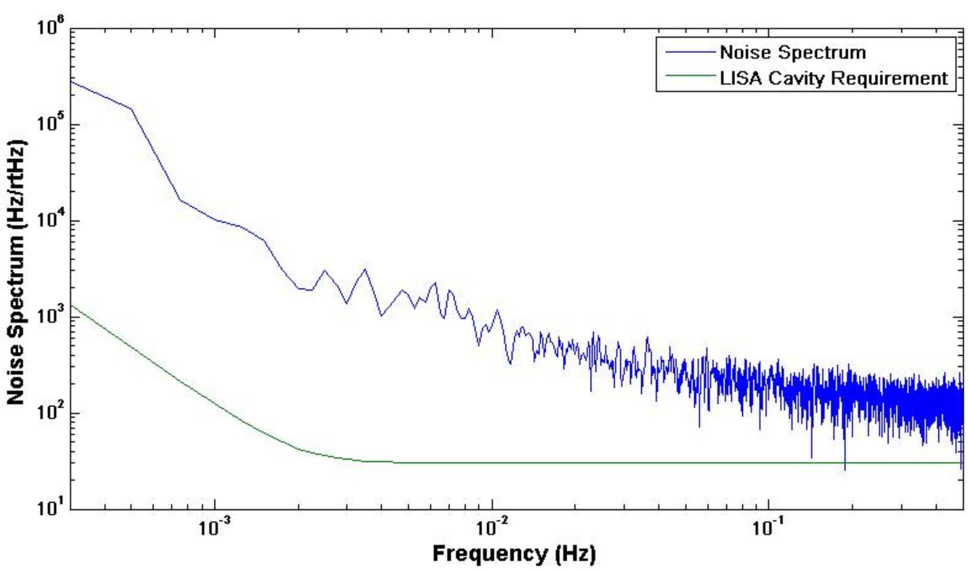

Figure 11: Noise spectrum without voltage applied.

When comparing the noise spectrum of the cavities with and without a voltage, we can see that they are very close (*See Figure 12*), but that there is more noise with a voltage.

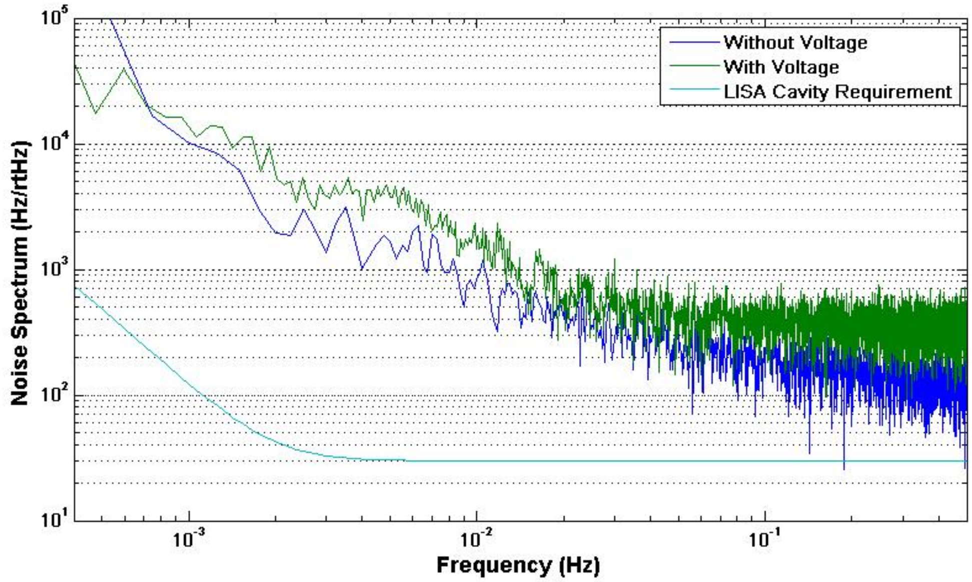

Figure 12: Noise spectrum with (green) and without (blue) voltage.

We can infer our voltage noise from our noise spectrum. For example, at 1 Hz our noise spectrum is about 500 Hz/ $Hz^{1/2}$, and when divided by our PZT coefficient, 2.0 MHz/V, we get a voltage noise of .25 mV/ $Hz^{1/2}$. When applying this formulation to our entire spectrum, we can see a similar result (*figure 13*).

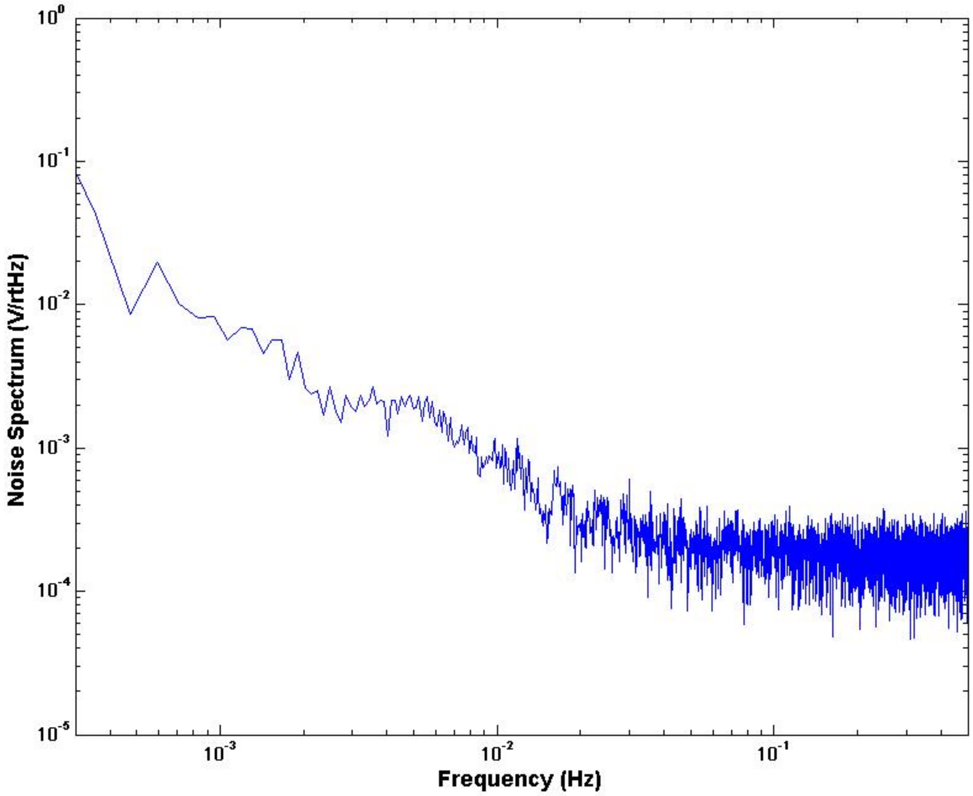

Figure 13: Inferred voltage noise.

Discussion

These preliminary results are most likely limited by noise sources other than the cavity, such as electronics and problems with alignment. Our low frequency noise may be due to temperature fluctuations. In the future, these fluctuations will be checked. We also need to reduce the noise brought on by our PZT. One solution to creating less noise from our PZT would be the have the Zerodur material be 10 times longer than the PZT material. This change would reduce our noise by a factor of 10 getting closer to where we would want it to be. Also in the future, we will need a voltage supply with a lower noise spectrum to meet the required 15 μV/ Hz$^{1/2}$.

Acknowledgments

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