A parametric amplification measurement scheme for MEMS in highly damped media

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ABSTRACT

Micro-electro-mechanical systems (MEMS) oscillators have been successfully used to probe properties of low viscosity fluids, and focus is now moving to using these devices in the highly damped, low Q regime. MEMS oscillators have been shown to be sensitive to parametric amplification. When used as part of a measurement scheme, we expect the device will produce a mechanically amplified signal before undergoing electrical amplification to preserve good signal-to-noise ratio. Preliminary theoretical analysis and circuit design is presented. The next steps to be taken are then discussed.

I. INTRODUCTION

The use of micro-electro-mechanical systems (MEMS) oscillators as low viscosity fluid probes has proven to be a promising endeavor, and there is currently effort towards expanding their use in more damped media. The resonance modes of these devices have been shown to have high quality factor (Q) in vacuum and are responsive to attempts to increase their sensitivity, particularly with the use of parametric amplification, which makes them excellent candidates for use in a highly damped regime [1,2].

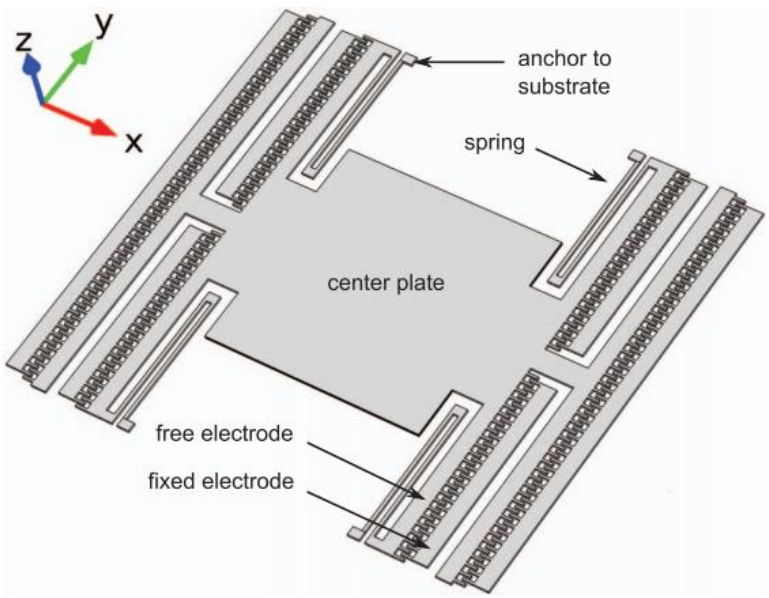

Figure 1: An image of the previous generation of MEMS device. It is a type of comb-drive oscillator, put into motion by an AC voltage across the electrodes, creating a capacitive driving force. The center plate is connected to four anchors by serpentine springs but is otherwise free to move. The electrodes are in pairs of fixed, also attached to the substrate, and free, which move with the plate. Image adopted from ref. [2].

Already used in the Lee research group are approximately planar 200×200 μm^2 capacitive MEMS oscillators (see Fig. 1). The device is operated by sweeping through its resonance, particularly its shear mode, and extracting information such as viscosity from its resonance peaks [4]. Thus far, this device has been characterized in both air and helium for use in the study of quantum turbulence in superfluid ^{4}He as well as the surface excitations in superfluid ^{3}He down to the sub-millikelvin range [1,3,4]. Experimental results show promising agreement with theory [3,4].

The phenomenon of parametric resonance occurs when a parameter of an oscillatory system is modulated at twice the frequency of the natural frequency of the system [5]. To illustrate this, consider the simple case of a swing whose length is modulated with time in Fig. 2. It can be easily shown by invoking conservation laws that this technique mechanically amplifies the amplitude while also serving as the driving force of the system [6]. Adopting notation from ref. [5], the general equation of motion for a parametric harmonic oscillator is

$$\ddot{x} + \omega^2(t)x = 0,$$

where $\omega(t)$ depends on the problem. Supposing it is periodic with frequency γ, then it follows that

$$\omega\left(t + \frac{2\pi}{\gamma}\right) = \omega(t).$$

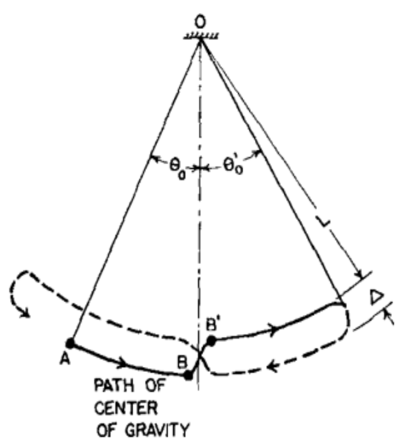

Figure 2: The modulation of the length at twice the natural frequency of the swing induces a gain in amplitude. Image adopted from ref. [6].

Thus, given the two solutions of the equation of motion, $x_1(t)$ and $x_2(t)$, there must also exist solutions $x_1(t+\frac{2\pi}{\gamma})$ and $x_2(t+\frac{2\pi}{\gamma})$, meaning these solutions must be linear combinations of $x_1(t)$ and $x_2(t)$. It is possible to choose $x_1(t)$ and $x_2(t)$ such that

$$x_1(t) = \mu_1^{\frac{t\gamma}{2\pi}} f_1(t)$$

$$x_2(t) = \mu_2^{\frac{t\gamma}{2\pi}} f_2(t)$$

where $f_1(t)$ and $f_2(t)$ are periodic functions with frequency γ and μ_1 and μ_2 are constants. From the equation of motion, the condition

$$\mu_1\mu_2 = 1$$

gives the two solutions as

$$x_1(t) = \mu^{\frac{t\gamma}{2\pi}} f_1(t)$$

$$x_2(t) = \mu^{-\frac{t\gamma}{2\pi}} f_2(t).$$

Together, these two equations represent an exponential increase in displacement of the oscillatory system after some time [5].

Of particular interest is the case where $\omega^2(t) = \omega_0^2[1 + h\cos(2\omega t)]$. In this case, the equation of motion is

$$\ddot{x} + \omega_0^2[1 + h\cos(2\omega_0 t)]x = 0.$$

This is a Mathieu equation that visually represents an oscillatory system with natural frequency ω_0 modulated by a periodic function with frequency $2\omega_0$ and amplitude $\omega_0^2 h$. It can be shown that the modulating periodic function creates the greatest gains in amplitude when $\omega = \omega_0$ [1].

Parametric amplification is similar in theory to parametric oscillation. This method involves the supplementary modulation of some parameter of an oscillatory system at an amplitude less than some constant, discussed in ref. [7]. A common way of achieving this is to modulate the spring constant in a damped spring-mass system by applying the gradient of an additional sinusoidal force[6]. Here, the equation of motion takes the form

$$m\ddot{x} + \frac{m\omega_0}{Q}\dot{x} + \left[k_0 + k_p(t)\right]x = F(t)$$

where $\frac{m\omega_0}{Q}$ is the damping coefficient, Q is the quality factor, k_0 is the spring constant of the unmodulated system, $k_p(t)$ is the time-dependent modulation of the spring constant, and $F(t)$ is the driving force. When $F(t)$ is of the form $F(t) = F_0 \cos(\omega_0 t + \varphi)$, φ is the relative phase between the driving force and spring constant modulation, and $k_p(t)$ is of the form $k_p(t) = \Delta k \sin(2\omega_0 t)$, it is shown in ref. [7] that the displacement of this system is

$$x(t) = F_0 \frac{Q\omega_0}{k_0}\left[\frac{\sin\varphi}{1 - \frac{Q\Delta k}{2k_0}}\cos(\omega_0 t) + \frac{\cos\varphi}{1 + \frac{Q\Delta k}{2k_0}}\sin(\omega_0 t)\right]$$

This method results in a mechanical amplification of the signal and an increased quality factor Q without sacrificing a good signal-to-noise ratio, a result that has been experimentally verified [1]. Gains of over 100 have been reported [1].

As shown, the current MEMS devices are promising probes at low temperatures and in low viscosity environments, and there is motivation to expand the use of these devices into the heavily damped regime [3,4]. The purpose of this project is to fabricate a new generation of MEMS oscillators capable of accurately probing highly damped media and design a measurement scheme making use of parametric amplification for this device. At present, a measurement scheme has been designed and is ready to be used in conjunction with the new MEMS device. It is expected that this will allow the MEMS device to be operated with an enhanced signal to noise ratio. This scheme will be tested in damped fluids such as water and glycerin.

II. THEORY

A. Scheme without parametric amplification

The measurement scheme currently in use consists of a single AC voltage source of the form $V_f = A\cos(\omega t)$ applied across the electrodes of the driving side of the device (see Fig. 3). This creates a potential energy $U = \frac{1}{2}CV^2$ and, thus, an attractive capacitive driving force

$$F = \frac{\partial U}{\partial x} = \frac{1}{2}\frac{dC}{dx}V_f^2 = \frac{1}{2}\beta V_f^2$$

where β is the transduction factor [2].

On the detection side, a DC voltage source, V_b, is applied to the MEMS, which causes a positive charge $Q_0 = C_2V_b$ to accumulate on the detection side fixed electrode. The free

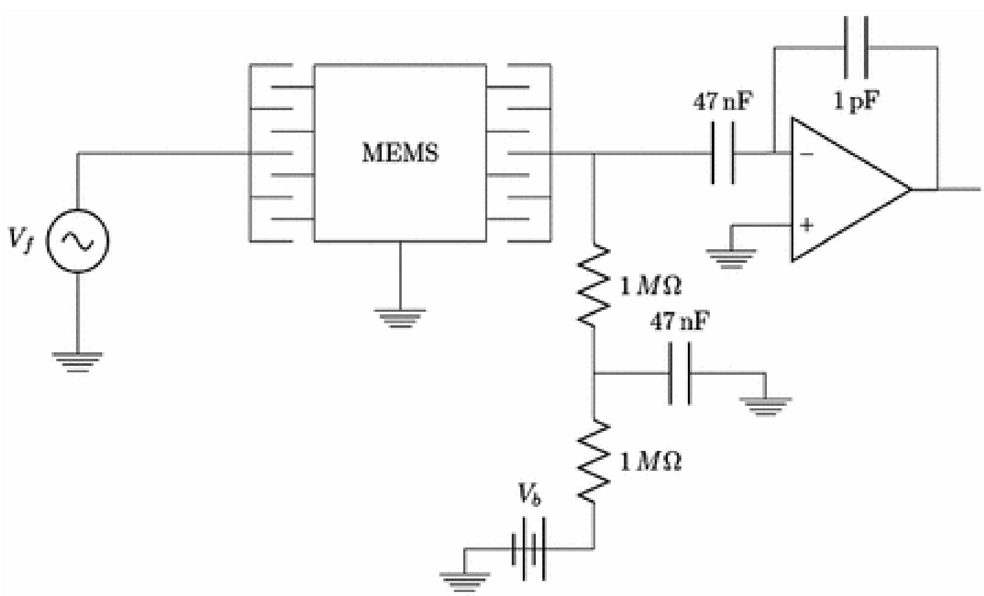

Figure 3: A schematic diagram of the current measurement scheme. A single AC voltage source on the driving side of the MEMS and a single DC voltage source on the detection side drive the device. The charge on the driving side of the MEMS is detected and amplified by a charge sensitive amplifier. Image adapted from ref [8].

electrode, connected to ground, then has a charge $-Q_0$. The application of V_f creates another charge,

$$Q = CV_f = (C_1 + \delta C)V_f = Q_1 + \delta Q$$

which changes with time as the driving force F causes a change in the distance between capacitor plates and thus a change in capacitance. The total charge on the detection side is then

$$Q_{total} = Q_0 + Q_1 + \delta Q.$$

A charge sensitive amplifier with amplification factor α outputs a voltage of the form

$$V_0 = \alpha Q_{total} = V_x + iV_y = \frac{\alpha\beta^2 V_f^2 V_b}{2m} \frac{(\omega_0^2 - \omega^2) + i(\frac{\gamma}{m}\omega)}{(\omega_0^2 - \omega^2)^2 + (\frac{\gamma}{m}\omega)^2}$$

where V_x and V_y are the in- and out-of-phase components, m is the mass of the MEMS, ω_0 is the natural frequency, ω is the driving frequency, and γ is the damping coefficient of the media, as shown in ref [4]. It is then possible to fit experimental results to these in-phase and out-of-phase curves and extract information such as the damping coefficient.

B. Scheme with parametric amplification

In the new scheme, two voltage sources, $V_1 = A\cos(\omega t)$ and $V_2 = B\cos(2\omega t)$ are combined and fed into the MEMS (see Fig. 4) so that

$$V_{total} = V_1 + V_2 = A\cos(\omega t) + B\cos(2\omega t).$$

The new scheme is similar to that of the previous section. A current sensitive amplifier will be used to detect the change in current on the detection side of the MEMS device. Further

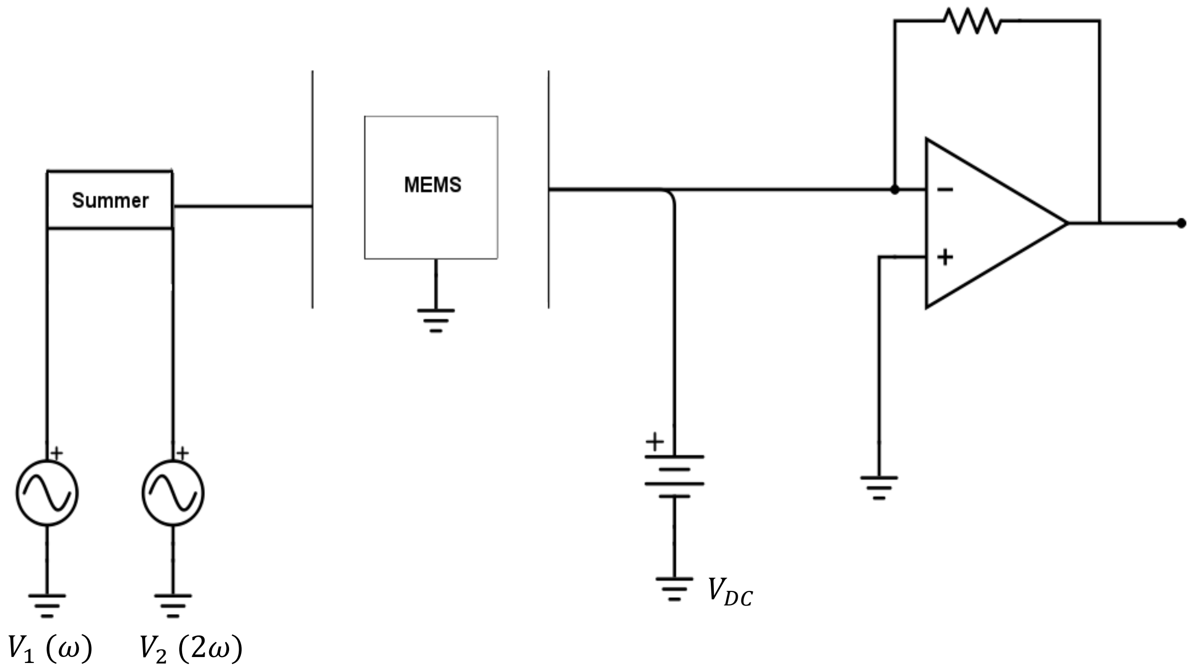

Figure 4: A schematic diagram of the measurement scheme involving parametric amplification. The setup is the same as in figure with the exception of an additional AC voltage source with frequency 2ω summed with the driving voltage with frequency ω and the use of a current-sensitive amplifier.

theoretical analysis of this scheme is still to be done and will rely heavily on the discussion in the introduction and previous section.

III. IMPLEMENTATION

Both the driving and modulating voltages will be provided by a Tektronix AFG3252 dual-channel arbitrary waveform generator. The AC driven side will be modulated at twice the driving frequency to produce parametric amplification. The signal on the DC bias side is produced by a 16 bit bipolar DAC. Programs will be designed to detect the amplified signal and analyze the results. Once complete, it will be possible to test the MEMS not only in various media but also at various pressures and temperatures.

IV. CONCLUSION

The preliminary setup for a parametric amplification measurement scheme for a new generation of MEMS oscillator is complete. Once the new device is ready to be implemented, testing can officially begin. Further theoretical analysis with emphasis on the effects of the parametric amplification will be done, and experimental demonstration of shear mode displacement amplification is expected. Future plans involve testing the device's ability to accurately measure the viscosity of fluids such as water and glycerin.

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