PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part A, 21 August 2007, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

6. Each problem is worth 10 points.

7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

A1. Consider the annihilation operator a and the creation operator $a^\dagger$ which satisfy the commutation relation

$$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1$$

where 1 is the unit operator. Let the number operator be

$$N \equiv a^\dagger a .$$

(a) (*2 points*)

(i) Reexpress $(a^\dagger)^2 a^2$ as a function of N.

(ii) Reexpress $(a^\dagger)^3 a^3$ as a function of N.

(b) (*2 points*) Find a general expression for $(a^\dagger)^n a^n$ as a function of N.

Let $|n\rangle$ be eigenvectors of the number operator N. In particular

$$N|n\rangle = n|n\rangle ,$$

showing that n is the eigenvalue associated with the eigenvector $|n\rangle$.

(c) (*2 points*) Given that N is Hermitian,

(i) Show that $|n\rangle$ and $|m\rangle$ are orthogonal for $n \neq m$. Hereafter assume that each $|n\rangle$ is normalized to unit length.

(ii) Based on the answers to (a), (b), and part (i) of (c), show that the eigenvalues n lie in the set $\{0, 1, 2, 3, \ldots\}$

Let $|0\rangle$ denote the ground state for which $N|0\rangle = 0$.

(d) (*2 points*)

(i) Express the state $|1\rangle$ as a function of a and $a^\dagger$ acting on the state $|0\rangle$

(ii) Express the state $|2\rangle$ as a function of a and $a^\dagger$ acting on the state $|0\rangle$.

(e) (*2 points*) Express the state $|n\rangle$ as a function of a and $a^\dagger$ acting on the state $|0\rangle$.

A2. An electron moves in a circular orbit around a nucleus of charge Ze.

(a) (*2 points*) What is the electron speed v as a function of the radius r of the orbit? Treat the motion of the electron according to the laws of classical mechanics.

(b) (*2 points*) Let us adopt cylindrical coordinates (ρ, z, ϕ) such that the electron is moving in the $z = 0$ plane in the $\hat{\phi}$-direction. $\hat{z}$ and $\hat{\phi}$ are related by the right-hand rule. A uniform magnetic field $\vec{H}_1 = H_1(t)\hat{z}$ is turned on, inducing an electric field of the form $\vec{E}_1 = E_1(\rho, t)\hat{\phi}$. Give an expression for $E_1(\rho, t)$ in terms of $H_1(t)$.

(c) (*3 points*) Let us assume that $H_1(t)$ varies from zero to an infinitesimally small positive value dH. What is the change in the speed of the electron? Does it speed up or down?

(d) (*3 points*) What is the change in the radius of the electron orbit?

A3. An eccentric professor proposes to explain the anomalous rotation curves (rotational velocity as a function of radial distance from galactic center) of galaxies in terms of gravitationally attractive strings of matter (essentially, one dimensional lines of mass) passing through the center of each galaxy and oriented perpendicular to the disk of the galaxy.

(a) (*3 points*) Starting from Newton's Law of gravitation, calculate the gravitational force on a particle of mass M a distance R from an infinitely long straight gravitationally attractive "string" of mass per unit length μ.

(b) (*2 points*) Use the result from part (a) to calculate the velocity of a particle of mass M in circular orbit about the string at a distance R from it.

(c) (*1 point*) Some galaxy is observed to have a rotation curve which asymptotes to a constant v_{obs} m/s for its most distantly observable parts. What mass per unit length would be required of a string which could account for the observed rotation curve of the galaxy?

(d) (*2 points*) In special relativity, energy and mass are related by Einstein's most famous formula. An important fact in physics is that energy per unit length and tension have the same physical dimensions. What would be the speed of sound in a string if it were maintained at a tension equal to its energy per unit length?

(e) (*2 points*) One might question whether the string in part (d) could be stable. In your understanding of physics, what could it mean to say that such a string is unconditionally stable in response to a small, localized, sideways displacement?