Student ID Number: __________

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA

Part D, 22 August 2007, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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D1. An object of mass m orbits another object in a central force described by $f(r) = -k^2 m/r^\alpha$, with $\alpha > 1$. Assume that the orbiting object is much less massive than the second object.

(a) (*2 points*) Assume that the motion is circular, with radius r. What is the total energy of the object, assuming that the potential energy is defined to be 0 at $r = \infty$? Express your answer in terms of r, k and α.

(b) (*2 points*) Assume that the motion is circular, with radius r. If the object receives a sudden kick (i.e. sudden increase) in velocity Δv, what is the minimum value of Δv needed to completely escape from the vicinity of the other object? What is minimum value of Δv needed to completely escape from the vicinity of the other object no matter what direction the kick occurs? Express your answers in terms of r, k and α.

(c) (*3 points*) Assume again that the motion is circular, with radius r. If the object is perturbed slightly, what is the ratio of the oscillation frequency to the orbital frequency in terms of k, r, α? For what values of α is the motion periodic? For cases where the motion is not periodic, describe qualitatively what happens when the object is slightly perturbed from its orbit.

(d) (*3 points*) Assume an inverse square law ($\alpha = 2$) for the central force. If the object is orbiting in a circle of radius r around the sun, find the minimum velocity kick needed to reach radius nr ($0 < n < \infty$), expressing your answer in terms of r, k and n. Note that the new orbit is an ellipse with closest distance at r and maximum distance at nr.

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D2. (a) (*5 points*) The constant-volume heat capacity C_V of a solid is given by

$$C_V = aT + bT^3,$$

where T is the temperature, and a and b are constants. Give the entropy of the solid as a function of the temperature.

(b) (*5 points*) The entropy S of another material is given by

$$S = R\left[\ln(\frac{U}{U_0}) + \ln(\frac{V}{V_0})\right],$$

where U is the energy and V the volume. R, U_0, and V_0 are constants. Give the volume of this material as a function of temperature and pressure.

PRELIMINARY EXAMINATION

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D3. In a so-called LS (or Russell-Saunders) angular momentum coupling scheme for atoms, one combines separately the orbital angular momenta into total orbital angular momentum, with quantum number L, and the electron spins in total spin angular momentum, with quantum number S. Next, these momenta are coupled to obtain the total electron angular momentum, with quantum number J, according to

$$J = L + S, L + S - 1, L + S - 2, \ldots |L - S|.$$

Thus, within this scheme, atomic energy levels (also called fine-structure levels) are characterized by their J, L, and S values. In the absence of a magnetic field these states are $(2J + 1)$-fold degenerate.

If, in addition, the atomic nucleus has a spin quantum number I, the so-called electron-nuclear (or hyperfine) levels arise, characterized with quantum numbers (F, J, L, S) with

$$F = J + I, J + I - 1, J + I - 2, \ldots |J - I|.$$

An atom has two electrons outside closed shells in the configuration $4s3d$: one electron occupies a $4s$ orbital $(l = 0)$, and one occupies a $3d$ orbital $(l = 2)$. We speak of an atom with *two optically active* electrons. Its nucleus has a spin $I = 1/2$.

(a) (*3 points*) How many distinct hyperfine states does this atom have?

(b) (*1 point*) What are the total orbital angular momenta quantum numbers L?

(c) (*1 point*) What are the total spin angular momenta quantum numbers S?

(d) (*1 points*) What are the total electron angular momenta quantum numbers J?

(e) (*2 points*) What are the total hyperfine quantum numbers F?

(f) (*2 points*) The energy of a hyperfine level is shifted relative to that of an associated fine-structure level by

$$\Delta \bar{E} = C[F(F + 1) - I(I + 1) - J(J + 1)].$$

What is the $\Delta \bar{E}$ for each (F, J) state?