Student ID Number: ________

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part A, August, 2014, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. A free particle of energy E and mass m traveling in the positive x direction impinges upon a square potential barrier. The potential energy at the top of the barrier is $V(V > E)$ and spans the region $0 \leq x \leq D$. The barrier divides space into three regions:

- before the barrier ($x < 0$)
- within the barrier ($0 \leq x \leq D$)
- after the barrier ($D < x$).

(a) [**1 point**] Write the relevant Schrœdinger equations for the three regions.

(b) [**1 point**] Write the wave function of the particle in the three regions.

(c) [**1 point**] What boundary conditions must the wave function satisfy at the edge of a finite potential barrier?

(d) [**2 points**] For the problem at hand, translate these into conditions on the wave function parameters.

(e) [**3 points**] Find the reflection coefficient for the particle, *i.e.*, the probability that it will be found traveling in the opposite direction after reaching the barrier.

(f) [**2 points**] Find the transmission coefficient for the particle, *i.e.*, the probability that it will be found on the other side of the barrier ($x > D$).

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

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A2. The Hamiltonian of a system is

$$\hat{H} = \varepsilon \vec{\sigma} \cdot \hat{n} = \varepsilon(\sigma_x \sin\theta \cos\phi + \sigma_y \sin\theta \sin\phi + \sigma_z \cos\theta),$$

where ε is a real constant and $\sigma_x \sigma_y \sigma_z$ are Pauli matrices.

(a) **[5 points]** Examine whether or not the following vectors are eigenfunctions of the Hamiltonian. (Answer by guessing is not acceptable.)

$$|1\rangle = \begin{pmatrix} e^{-i\phi/2} \cos\frac{\theta}{2} \\ e^{-i\phi/2} \sin\frac{\theta}{2} \end{pmatrix} \quad \text{and} \quad |2\rangle = \begin{pmatrix} -e^{-i\phi/2} \sin\frac{\theta}{2} \\ e^{-i\phi/2} \cos\frac{\theta}{2} \end{pmatrix}$$

(b) **[5 points]** Calculate $\hat{U}^\dagger \hat{H} \hat{U}$ where

$$\hat{U} = \begin{pmatrix} e^{-i\phi/2} \cos\frac{\theta}{2} & -e^{-i\phi/2} \sin\frac{\theta}{2} \\ e^{-i\phi/2} \sin\frac{\theta}{2} & e^{-i\phi/2} \sin\frac{\theta}{2} \end{pmatrix}.$$

PRELIMINARY EXAMINATION

Department of Physics
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A3. A linear oscillator with mass m and angular frequency ω is described by the following wave-function

$$\Psi(x,t) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left\{ i\frac{P_o x}{\hbar} - \frac{m\omega}{2\hbar}(x - X_o)^2 - i\left[\frac{PoXo}{2\hbar} + \frac{\omega}{2}\right]\right\}$$

where $\hbar$ is Plank constant and Po and Xo are the time-dependent mean values of the momentum and coordinate probability distributions

$$X_o(t) = \sqrt{\frac{\hbar}{m\omega}}\sin(\omega t), \quad P_o(t) = \sqrt{\hbar m\omega}\cos(\omega t)$$

(a) **[1 point]** What is the shape and root-mean-square (rms) of the coordinate probability density distribution?

(b) **[2 points]** Find the root-mean-square (rms) of the momentum probability density distribution.

(c) **[3 points]** Find the average energy of the oscillator.

(d) **[4 points]** Find the probability for the oscillator to be in the ground state.