DEPARTMENT OF PHYSICS UNIVERSITY OF FLORIDA Part B, August, 2014, 14:00–17:00

Instructions

- 1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
 - (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
 - (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
 - (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
 - (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
 - (e) Each problem is worth 10 points.
 - (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."

DO NOT OPEN EXAM UNTIL INSTRUCTED

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B1. A uniformly charged solid spherical shell of radius R carries total surface charge Q, and is set spinning with angular velocity ω about the z axis passing through its center as shown in figure.



- (a) [2 points] What is the magnetic dipole moment of the shell?
- (b) [3 points] Find the magnetic field at the center of the sphere.
- (c) [2 points] Find the approximate vector potential at a point (r, θ) where $r \gg R$.
- (d) [3 points] Find the magnetic field at (r, θ) where $r \gg R$.

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B2. A popular physics trick involves the use of a capacitor that appears to "magically" charge itself. A capacitor with a large surface area and some dielectric material between the plates is originally charged to a high voltage, and subsequently disassembled and thoroughly grounded, removing all charge from the capacitor's plates. The capacitor is subsequently reassembled, and when touched will cause a large spark to jump to the unsuspecting finger (showing that there was a large amount of charge left in the capacitor). This phenomena is due to the "real world" effect of imperfect contact between the capacitor plates and dielectric surfaces, but is fairly straightforward to analyze.

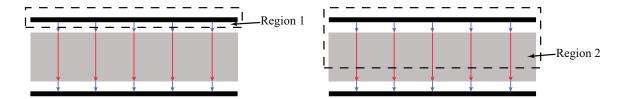
For this problem, you need to work in SI units, where the electric field of a point charge is given by $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$, and the electric field of a large planar conductor with area A and charge q is $|E| = \frac{q}{2\epsilon_0 A}$.

- (a) [1 point] A parallel plate capacitor is constructed from two parallel sheets with area A, separated by a distance d. What is the capacitance C of this system? (Obtain an analytic expression for C.)
- (b) [2 points] If a dielectric is placed in between the plates, with $\kappa = \frac{\epsilon}{\epsilon_0} = 5$ and thickness b = d 2g (where g is the gap between the dielectric and the plates) show that the new value of the capacitance is given by $C = \frac{\kappa \epsilon_0 A}{(d-2g) + 2\kappa g}$



- (c) [1 point] How much charge q_1 is needed to be placed on the plates of a 100 pF capacitor with a d=1mm plate separation (containing no dielectric) to achieve a voltage of $\Delta V=25000$ V? What is the electric field in the capacitor? (Obtain numerical values for q_1 and E.)
- (d) [1 point] If the breakdown field in air is 5×10^7 V/m, what is the maximum charge that the capacitor (with no dielectric) can hold? What is the maximum voltage that can be reached? (Obtain numerical values for q_{max} and V_{max} .)

(e) [1 point] If a dielectric material is added to the capacitor as show below, a charge of q_1 is placed on the capacitor plates, and a charge q_2 placed directly on the dielectric's top/bottom surfaces, use the shown gaussian surfaces to calculate the electric field in regions 1 and 2. (Obtain an analytic expression for E_1, E_2 .)



(f) [1 point] Show that the voltage across the entire capacitor is

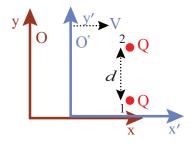
$$\frac{1}{\epsilon_0 A} \left(2gq_1 + \frac{1}{\kappa} (q_1 + q_2)(d - 2g) \right).$$

(Hint: Use the results of the previous part to calculate the voltage across regions 1 and 2.)

- (g) [1 point] Using the results of parts (d) and (f), calculate how much charge q_2 is needed on the dielectric to obtain a desired total voltage of ΔV that is larger than the ΔV_{max} obtained in part (c). (Obtain a analytic expression for q_2 .)
- (h) [1 **point**] If the average gap between the dielectric and the capacitor plates is $g = 10\mu\text{m}$, use the result from part (g) to determine how much charge q_2 is needed on the dielectric to get a total voltage of $\Delta V = 25000 \text{ V}$. (Obtain a numerical value for q_2 .)
- (i) [1 point] If all the charge q_1 was removed from the plates, and q_2 remained on the dielectric, what is the remaining electric field energy U in the capacitor? (Obtain a numerical value for U.)

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B3. Two particles with equal charge $Q = 3\mu C$ are at rest in the O'-frame (particle frame) separated by a distance $\Delta y' = d = 2$ m, with $\Delta x' = \Delta z' = 0$, as shown in the figure. The two particles are moving with a constant speed V = 0.8c in the x-direction in the O-frame (lab frame), where c is the speed of light in a vacuum. The O'-frame lies in the xy-plane of the O-frame with y = y' and z = z', (Do this problem relativisticly correct!)



- (a) [1 point] What is the magnitude (in milliN) and direction of the electric force that particle 1 exerts on particle 2 in the particle frame?
- (b) [1 point] What is the magnitude (in milliN) and direction of the magnetic force that particle 1 exerts on particle 2 in the particle frame?
- (c) [2 points] What is the magnitude (in milliN) and direction of the electric force that particle 1 exerts on particle 2 in the lab frame?
- (d) [2 points] What is the magnitude (in milliN) and direction of the magnetic force that particle 1 exerts on particle 2 in the lab frame?
- (e) [2 points] What is the magnitude (in milliN) and direction of the total electromagnetic force that particle 1 exerts on particle 2 in the lab frame?
- (f) [2 points] Plot the ratio, R, of the magnitudes of the total force that particle 1 exerts on particle 2 in the lab frame and particle frame, R = F/F', versus $\beta = V/c$ from 0 to 1, where V is the relative velocity of between the two frames.