Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, August, 2015, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. **(Thorn)** The electron is a spin 1/2 fermion with mass. In this problem we consider a system containing two electrons.

(a) [**3 points**] Let $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$ be the total spin of the two electron system. List the allowed eigenvalues of $\boldsymbol{S}^2$ and S_z.

(b) [**5 points**] Introducing the spin basis states $|m_1, m_2\rangle$ which are eigenstates of the spin components S_z^1, S_z^2 of the two electrons, construct the eigenstates for each set of eigenvalues listed in part a).

(c) [**2 points**] Now imagine the two electrons are the electrons of the Helium atom. Ignoring spin dependent couplings and treating electron repulsion as a perturbation, let one electron be in the $nlm = 100$ eigenstate of H_0, and let the other be in the $nlm = 200$ eigenstate (so that they are in two different s-states). At zeroth order all of the different spin states of the two electrons will be degenerate. Without doing any calculations, argue how the Coulomb repulsion between the electrons will split the energies of the different spin states, identifying the ordering and degeneracy of each split level. Explain your reasoning.

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DEPARTMENT OF PHYSICS
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A2. **(Stanton)** An unperturbed Hamiltonian is given by:

$$H^0 = \epsilon_0 \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix}.$$

At $t = 0$, the system is in the *ground state* and then a perturbation is turned on:

$$H'(t) = \epsilon_0 \begin{pmatrix} 0 & 0 & F_t \\ 0 & 0 & 0 \\ F_t & 0 & 0 \end{pmatrix} \quad \text{where} \quad F(t) = \begin{pmatrix} 0 & t \leq 0 \\ 4(t/T) & 0 \leq t \leq T \\ 4 & t \geq T \end{pmatrix}.$$

1. Sudden Approximation

(a) [**2 points**] The "sudden approximation" occurs if T is small. Small compared to what?

(b) [**1 point**] In the sudden approximation, what is the state vector of the system at $t = T$?

(c) [**2 points**] What is the probability of being in the *new* ground state of the system at $t = T$?

(d) [**1 point**] What is the probability of being in the first excited state of the new system at $t = T$?

(e) [**1 point**] What is the probability of being in the second excited state of the new system at $t = T$?

2. Adiabatic Approximation

If T is large, so the perturbation occurs slowly, then one can use the "adiabatic approximation".

(f) [**2 points**] If the system starts in the ground state for $t < 0$, and the perturbation is turned on adiabatically, then what is the state of the system at $t = T$? (Do not worry about phase).

(g) [**1 point**] What is the probability of being in the new ground state of the system at $t = T$?

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A3. **(Konigsberg)** A spin 1/2 electron with magnetic moment μ is placed in a uniform magnetic field $\vec{B} = B\hat{k}$ in the positive z-direction. The intrinsic spin of the electron is pointed along the positive x-direction at $t = 0$.

(a) [**2 points**] Write down the Schrodinger equation for the two component wave function for the electron at rest.

(b) [**3 points**] Find the corresponding time-dependent wave function.

(c) [**3 points**] Calculate the expectation values $< S_x(t) >, < S_y(t) >, < S_z(t) >$.

(d) Find the probability as a function of time that the intrinsic spin will be pointed along:

- [**1 point**] the positive z-direction
- [**1 point**] the positive x-direction

This may be useful:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

$$| S_z^+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} | S_x^+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2}$$