Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, August, 2015, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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B1. **(Tanner)**

This problem is about the electromagnetic properties of a plasma, consisting of mobile electrons and of much less mobile ions. Several physical systems, such as pure metals at low temperatures, superconductors, and the ionosphere could be described as such a plasma.

The plasma is nonmagnetic and there is no external magnetic field.

In the following, regard the ions as completely immobile and ignore them in calculating currents and polarizations. The plasma as a whole is electrically neutral so there is no volume charge density: $\rho_{\text{free}} = 0$.

(a) [**2 points**] Consider the response of the plasma to an electric field. The electric field at point $\vec{r}$ is given by

$$\vec{E} = \hat{x}E_0 e^{-i\omega t},$$

and applies a force to an elecron located at $\vec{r}$. Calculate the time-dependent velocity $\vec{v}$ of the electron at $\vec{r}$ from Newton's laws. It will be a function of the electric field E, the frequency ω, the charge e and the mass m of the electron. You may take the initial velocity as zero.

(b) [**1 point**] The plasma has a uniform electron (and ion) density (number per unit volume) n. Write equations for the current density $\vec{\jmath}$ and the conductivity σ at $\vec{r}$.

(c) [**2 points**] Now let the electric field be in plane-wave form:

$$\vec{E} = \hat{x}E_0 e^{i(kz-\omega t)},$$

where the wave vector $\vec{k}$ is taken to point in the $\hat{z}$ direction. (This makes no change in the solutions to the first two parts of the question.) Show that this field (along with a similar equation for the magnetic field and with the current density from part b) can be plugged into Maxwell's equations to convert them to algebraic equations.

(d) [**1 point**] What direction must the magnetic field have?

(e) [**2 points**] Now eliminate the fields from your results in part c, and find an equation for the wave vector k in terms of n, e, m, and ω. *Don't forget the displacement field.*

(f) [**2 points**] For some frequencies the wave is damped and for others it propagates without loss. Determine the ranges of ω where these two conditions apply.

Note: If I have a plane wave vector $\vec{G}$,

$$\vec{G} = \vec{G}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)},$$

then it is easy to show that

$$\nabla \cdot \vec{G} = i\vec{k} \cdot \vec{G}$$

$$\nabla \times \vec{G} = i\vec{k} \times \vec{G}$$

$$\frac{\partial \vec{G}}{\partial t} = -i\omega\vec{G}$$

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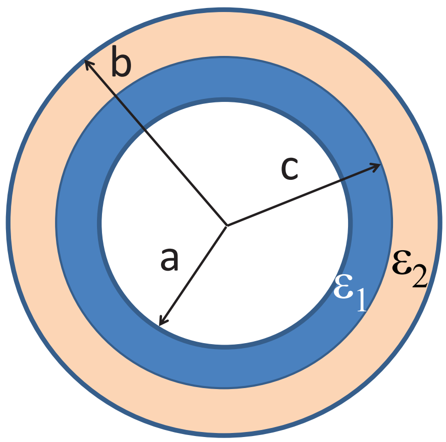
B2. **(Hirschfeld)** A t-dependent voltage $V(t) = V_0 \cos(\omega t)$ is applied to a capacitor, which consists of two concentric conducting spheres of radii a and b ($a < b$). The space in between the spheres is filled with two spherical shells made of *different* insulators, so that

$$\epsilon = \begin{cases} \epsilon_1 & \text{for } a < r < c \\ \epsilon_2 & \text{for } c < r < b. \end{cases}$$



(a) [**4 points**] Find the capacitance C.

(b) [**4 points**] Find the displacement current (direction and magnitude) in terms of C.

(c) [**2 points**] Find the magnetic field produced by the displacement current.

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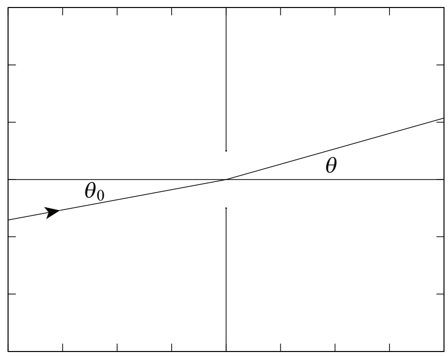
B3. **(Jim Fry)** A plane wave with wave vector at an angle θ_0 from the horizontal axis is incident on the aperture of dimension a.



(a) **[3 points]** At what angles θ in the x-y plane are there zeroes in the diffraction pattern?

(b) **[5 points]** What is the intensity as a function of angle θ?

(c) **[2 points]** What is the optimal aperture of a pinhole camera in which the film is placed a distance D from the aperture? Estimate for $D \approx 10$ cm.