Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics

University of Florida

Part A, August 18, 2016, 09:00–12:00

<u>Instructions</u>

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

<u>DO NOT OPEN EXAM UNTIL INSTRUCTED</u>

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, August 18, 2016, 09:00–12:00

A1. **(Lee)** Consider a weakly damped harmonic oscillator of a mass m driven by an external force $F(t) = mf_o \cos \omega t$. The equation of motion is given by

$$\ddot{x} + 2\beta\dot{x} + \omega_o^2 x = f_o \cos \omega t,$$

where x is the displacement of the oscillating object from the equilibrium position and ω_o is the natural frequency of the oscillator. The response of this oscillator after time $t \gg \frac{1}{\beta}$ is described by the particular solution,

$$x(t) = A \cos(\omega t - \delta).$$

(a) [**2 points**] Express A and δ using the given parameters in the problem.

(b) [**2 points**] Show that the maximum in amplitude A occurs at $\omega \approx \omega_o(1 - \frac{1}{4Q^2})$ for $\beta \ll \omega_o$. Here $Q = \omega_o/2\beta$ is called the quality factor.

(c) [**4 points**] There are three forces acting on the mass: (i) the spring force, $F_s = -m\omega_o^2 x$, (ii) the damping force, $F_d = -2m\beta\dot{x}$, and (iii) the driving force, $F(t)$. Among these, only the spring force is conservative, and the work done by the spring force in one cycle of oscillation should be zero. Calculate the work done by the damping force *and* the work done by the driving force *in one cycle* at a given driving frequency ω. What is the net work done by all the forces in one cycle?

(d) [**2 points**] At what frequency the driving force completely cancels the damping force? You may make a physical argument to answer this question.

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, August 18, 2016, 09:00–12:00

A2. **(Sikivie)**

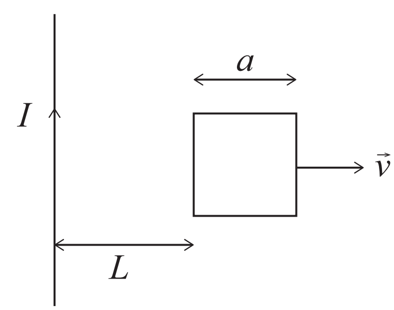
(a) [**2 points**] A current I runs along the z-axis from $-\infty$ to $+\infty$. What is the magnetic field $\vec{B}(\vec{r})$ everywhere due to that current?

(b) [**1 point**] *Electromotive force* is the name commonly given to the line integral of the electric field

$$\Phi_{\text{e.m.}}(t) = \oint \vec{dr} \cdot \vec{E}(\vec{r}, t)$$

along a closed loop. What are the units of so-called electromotive force?

(c) [**3 points**] A square circuit, made of thin wire, moves away from the z-axis as shown in the figure. Each side of the circuit has length a. The distance to the z-axis of the side closest to the z-axis is L. The velocity of the circuit relative to the z-axis is $v = \frac{dL}{dt}$. What is the magnitude of the electromotive force in the circuit due to the current I running along the z-axis?



(d) [**1 point**] Assuming $I > 0$, what is the direction (clockwise or counterclockwise) of the electromotive force in the square circuit due to the current I?

(e) [**3 points**] The electromotive force due to the current I produces a current i in the square circuit. What is i in terms of the information given above and the resistance R of the square circuit? Assuming that the speed v is constant in time, what order-of-magnitude condition must R satisfy in order that the electromotive force in the circuit due to the induced current i is negligible compared to the electromotive force due to I?

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part A, August 18, 2016, 09:00–12:00

A3. **(Woodard)**

(a) [**3 points**] Use the Biot-Savart law to compute the magnetic field at an arbitrary point $\vec{r} = (x, y, z)$ due to a wire segment (not a complete circuit) carrying current I which runs up the z axis from $z = -L/2$ to $z = +L/2$.

(b) [**2 points**] Whether or not you were able to solve part (a), let us agree to represent the x, y and z components of the answer in terms of three functions of x, y and z as follows:

$$\vec{B}_1(x, y, z) = \hat{x}C(x, y, z) + \hat{y}D(x, y, z) + \hat{z}E(x, y, z).$$

In this part you are asked to use these three functions to express the magnetic field of a closed wire carrying current I which forms a square of side length L. Suppose the square lies in the x-y plane, with its sides parallel to the x and y axes and its center at the origin. Suppose also that the current moves counterclockwise as seen from the positive z axis. What are the x, y and z components of the magnetic field at position $\vec{r} = (x, y, z)$ in terms of the three functions C, D and E? For example, you might answer that the x component is $E(x, y, z)+3C(z, y+L, x)$.

(c) [**1 point**] What is the magnetic dipole moment $\vec{m}$ of the square circuit described in part (b)? Note that you can answer this even if you could not get either part (a) or (b).

(d) [**2 points**] What is the dipole approximation for the magnetic field vector at position $vecr$ (valid at large r) for the square circuit in part (b)? Note that you can answer this (in terms an unknown dipole $\vec{m}$) even if you could not get parts (a-c).

(e) [**2 points**] Consider a positive charge q which moves counterclockwise with constant speed v in a circle of radius R in the x-y plane, centered on the current square. What is the magnitude and direction of the force exerted by the current square on the charge in the dipole approximation? Note that you can answer this even if you could not get any of the previous parts.