Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, August 18, 2016, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
   (e) Each problem is worth 10 points.
   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part B, August 18, 2016, 14:00–17:00

B1. **(Klimenko)** A thin metal sphere of radius a has charge q and the potential $\Phi = 0$. A second concentric sphere has radius $2a$. Space outside of the second sphere is filled with a medium with dielectric constant k (the medium permittivity is $\epsilon = \epsilon_0 k$, where ϵ_0 is the vacuum permittivity).

(a) [**2 points**] Find electric field as a function of the radius everywhere in space.

(b) [**2 points**] Find free charge of the system.

By carrying infinitesimal charges, the potential of the inner sphere is changed to V.

(c) [**2 points**] What is the variation of the potential of the second sphere?

(d) [**3 points**] Compute the work expended in charging the inner sphere to the potential V.

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS

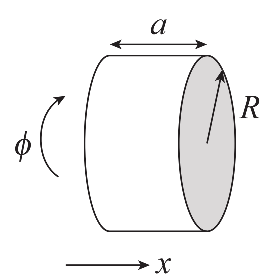
UNIVERSITY OF FLORIDA

Part B, August 18, 2016, 14:00–17:00

B2. **(Zhang)** Consider a particle of mass M and spin 1/2 moving on the surface of a strip that forms a circle (see figure). The strip has width a and length $2\pi R$ (the circle radius is R) with $R \gg a$. The Hamiltonian in the coordinate of (x, ϕ) is

$$\hat{H} = \frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} = \frac{\hbar^2}{2MR^2}\frac{\partial^2}{\partial \phi^2}.$$

The wave function of the particle $\psi(x, \phi)$ has the boundary conditions $\psi(0, \phi) = 0$ and $\psi(a, \phi) = 0$ and is periodic in ϕ.



(a) [**2 points**] Find the eigenenergies and the corresponding spatial wave functions (you do not need to normalize).

(b) [**2 points**] If there are two identical non-interacting particles with mass M and spin 1/2 on the strip, find the total energy of the lowest singlet ($S = 0$) state, and the corresponding wave function(s) (do not normalize). Express the wave function(s) in terms of the coordinates of the two particles (x_1, ϕ_1) and (x_2, ϕ_2) and the spin states $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$.

(c) [**2 points**] Find the total energy of the lowest triplet ($S = 1$) state and the corresponding wave function(s) (do not normalize).

(d) [**2 points**] Now add an exchange interaction between the two particles,

$$V_{\text{ex}} = -\lambda \left[|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{1}{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)(\langle\uparrow\downarrow| + \langle\downarrow\uparrow|) + |\downarrow\downarrow\rangle\langle\downarrow\downarrow| \right],$$

where λ is a real number. Express the corrections to all energy eigenvalues in terms of λ. You do not need to find the energy eigenvalues.

(e) [**2 points**] Under what condition for λ will the ground state total spin be 1?

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part B, August 18, 2016, 14:00–17:00

B3. **(Muttalib)** Consider a particle of charge e and mass m moving in a magnetic field $\vec{B}$, given by the Hamiltonian

$$H = \frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A})^2$$

where $\vec{p}$ is the momentum operator, $\vec{A}$ is the vector potential and c is the speed of light.

(a) [**2 points**] Obtain the velocity operator $\vec{v} = \dot{\vec{r}}$, where the overdot represents a time derivative. Show that the Hamiltonian can be written as $H = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)$. [Hint: The time derivative of an operator a is given by the commutator $\dot{a} = \frac{i}{\hbar}[H, a]$.]

(b) [**2 points**] Evaluate the commutator $[v_x, v_y]$. Show that it is proportional to the z-component of the magnetic field B_z. [Hint: $B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$.]

(c) [**2 points**] Assume tht the magnetic field is along the z-direction, *i.e.*, $B_x = B_y = 0$ and $B_z = B_0$. Use the commutators $[v_z, v_x]$ and $[v_z, v_y]$ to show that H can be written as the sum of two commuting operators, *i.e.*, $H = H_1 + H_2$ where $[H_1, H_2] = 0$.

(d) [**2 point**] Consider motion of the particle confined in the x-y plane, such that $v_z = 0$. Define $v_x = \omega X$, $v_y = \frac{1}{m}P$ and find ω such that $[X, P] = i\hbar$. Rewrite H in terms of X and P.

(e) [**2 point**] Use (d) to write down the energy levels of H. [Note: use well-known results; you do not need to solve the corresponding Schrodinger equation.]