Student ID Number: _____

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS UNIVERSITY OF FLORIDA Part D, August 19, 2016, 14:00–17:00

Instructions

- 1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
 - (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
 - (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
 - (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
 - (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
 - (e) Each problem is worth 10 points.
 - (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."

DO NOT OPEN EXAM UNTIL INSTRUCTED

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- D1. (Sullivan) A simple solid consists of molecules whose centers of mass are fixed at the lattice sites of a close packed lattice and each molecule having an electric dipole moment that interacts with its nearest neighbors.
 - (a) [3 points] If the mean polarization of each molecule is *P*, show that the configurational entropy per molecule is

$$S = \ln 2 - \frac{1}{2}(1+P)\ln(1+P) - \frac{1}{2}(1-P)\ln(1-P)$$

so that $S = \ln 2$ for P = 0 and S = 0 for $P = \pm 1$.

- (b) [4 points] If the interaction energy is $E = -AP^2$, show that, depending on the sign of A, there can be a transition from a disordered state (P = 0) to an ordered state $(P \neq 0)$. Calculate the transition temperature, T_c .
- (c) [3 points] Sketch the free energy functional as a function of P for (i) $T > T_c$, (ii) $T = T_c$, (iii) $T < T_c$.

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D2. (Maslov) Find the period of oscillations for a quartic one-dimensional oscillator

$$U\left(x\right) = \frac{1}{4}ax^4.$$

The amplitude of oscillations is 2A. You may need to use the numerical value of the following integral

$$C = \int_0^1 dy \frac{1}{\sqrt{1 - y^4}} = 1.31\dots$$

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- D3. (Avery) A planet of mass M and radius R has a moon of mass m and radius r in a circular orbit with a distance d between the centers of the two bodies.
 - (a) [5 points] If the moon's mass is neglected, its orbital period is calculated to be 5 days. Observations show that the true orbital period is 4 days. What is m/M?
 - (b) [3 points] Ignore the answer to (a) and now assume a different moon with $m \ll M$. If the moon is held together only by gravitational forces, show that if it is not to be torn apart by tidal forces (*i.e.*, an object on the moon's surface will not float away) then $d > R(2\rho_M/\rho_m)^{1/3} \simeq 1.26R(\rho_M/\rho_m)^{1/3}$, where ρ_M and ρ_m are the densities of the planet and moon, respectively. Assume the moon is not rotating.
 - (c) [2 points] If the moon's rotation is synchronously locked to its orbital period, show that $d > R(3\rho_M/\rho_m)^{1/3} \simeq 1.44R(\rho_M/\rho_m)^{1/3}$, for the moon not to be torn apart by tidal forces.