

Student ID Number: _____

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

Part A, August, 2017, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
 - (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
 - (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
 - (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
 - (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
 - (e) Each problem is worth 10 points.
 - (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *“On my honor, I have neither given nor received unauthorized aid in doing this assignment.”*

DO NOT OPEN EXAM UNTIL INSTRUCTED

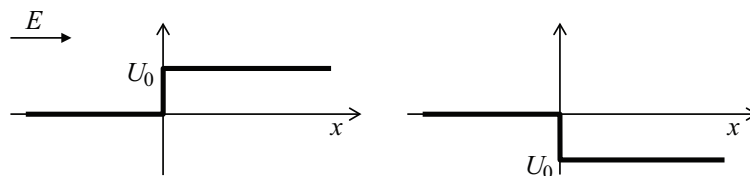
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- A1. (**Korytov**) Non-relativistic particles of mass m and kinetic energy E move freely from the left ($x = -\infty$) to the right and encounter a step in potential energy at $x = 0$, $U_0 \cdot \theta(x)$, where U_0 can be either positive or negative as shown in the figure. Assume $E > U_0$. Find reflection and penetration probabilities R and P for the particles. [5 points]



Further questions:

- (a) [1 point] Show explicitly that $R + P = 1$.
- (b) [1 point] In the case of the positive step and $E = U_0$, show what happens to R and P .
- (c) [1 point] In the case of a very deep negative step ($E \ll |U_0|$), show what happens to R and P .
- (d) [2 points] Finally, assume that $E \gg |U_0|$ and compare the reflection probabilities for the positive and negative steps in the lowest order of $\alpha = U_0/E$.

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A2. **(Cheng)** A particle of mass M moves in a 3D spherical harmonic oscillator potential $V(r) = \frac{1}{2}M\omega^2r^2$.

- (a) **[2 points]** Use the separation of variables in Cartesian coordinate to find the energy eigenvalues E_N in terms of the principal quantum number $N = 0, 1, 2, \dots$
- (b) **[2 points]** Find the normalized ground state wave function.
- (c) **[3 points]** Find the degree of degeneracy for the energy level E_N .
- (d) **[3 points]** For $l = 0$ states, there is no angular dependence of the wave function. In this case show that the radial solution can be mapped to the 1D half harmonic oscillator problem,

$$U(x) = \begin{cases} \infty, & x \leq 0; \\ \frac{1}{2}M\omega^2x^2 & x > 0. \end{cases}$$

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- A3. (**Hershfield**) The spin degree of freedom of a proton in a magnetic field, \vec{B} , is described by the Hamiltonian

$$H = -\gamma\vec{B} \cdot \vec{S},$$

where γ is the gyromagnetic ratio and \vec{S} is the spin operator, which can be written in terms of the Pauli spin matrices, $\vec{S} = (\hbar/2)\vec{\sigma}$. For your reference the Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

- (a) [**2 points**] At $t = 0$ the proton's spin is pointing in the negative x-direction. In other words, $S_x|\psi(t = 0)\rangle = -(\hbar/2)|\psi(t = 0)\rangle$. What is $|\psi(t = 0)\rangle$ in the basis of the Pauli matrices?
- (b) [**2 points**] Suppose that the magnetic field is in the z-direction, $\vec{B} = B\hat{z}$. What are the eigenstates and eigenvalues of the Hamiltonian, H , above?
- (c) [**3 points**] What is $|\psi(t)\rangle$ given the initial state at $t = 0$ of question (a) and the Hamiltonian of question (b)?
- (d) [**3 points**] What is the probability as a function of time that a measurement of the S_x yields $+(\hbar/2)$?