Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, August, 2017, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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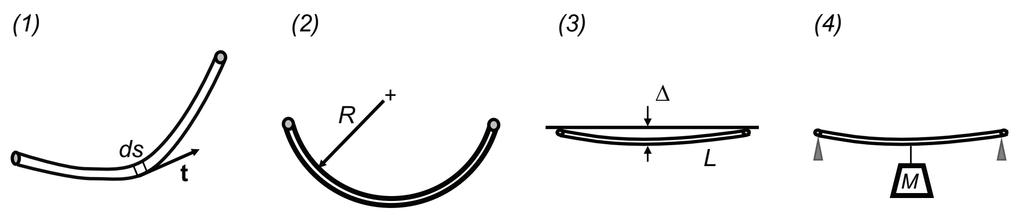
C1. **(Hagen)** Consider a flexible elastic rod that is bent slightly as in Figure (1). The elastic energy E stored per unit length s of the rod is given by:

$$dE/ds = \left(\frac{1}{2}\right)\kappa\left(\frac{d\hat{t}}{ds}\right)^2$$

where $\hat{t}$ is the local tangent unit vector at a position s along the length of the rod. The constant κ depends on the elastic modulus of the rod.



(a) [**3 points**] Suppose the rod is bent into a circular arc of radius R, as in Figure (2). Find the value of $\frac{dE}{ds}$ in terms of κ and R. (Note that you can express $\hat{t}$ in terms of a polar angle θ, where $s = R\theta$.

(b) [**3 points**] Suppose a rod of length L is bent very slightly so that its center is displaced by a small distance $\Delta \ll L$ as in Figure (3). Find the total elastic energy E of the rod in terms of L, κ, and Δ. (Suggestion: Find R.)

(c) [**4 points**] Now suppose that a rod with length L and negligible mass is supported horizontally by two points as shown in Figure (4). A mass M hangs from the center of the rod. Find the displacement Δ at the center of the rod when the system is at equilibrium. Give Δ in terms of M, g, L, κ.

You may assume that $\Delta \ll L$.

You may find these useful:

$$\sin\theta = \theta - \theta^3/3! + ... \qquad \cos\theta = 1 - \theta^2/2! + ...$$

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C2. **(Avery)** The earth gains mass due to interplanetary dust and meteorites falling steadily to its surface from all directions. Let the earth have mass M and radius R (from which its density ρ_0 can be determined) and rotational period T. Assume that the falling matter is isotropic with no net angular momentum and forms a layer (average density ρ_1) whose thickness h ($h \ll R$) increases at a rate of $\dot{h} = A\dot{m}$, where $\dot{m}$ is the rate of mass accretion and A is a constant.

(a) [**2 points**] Find the constant A in terms of R and ρ_1.

(b) [**6 points**] Find the change in rotational period (including sign) for a layer of thickness h and density ρ_1, expressing your answer in terms of T, R, h, ρ_0 and ρ_1.

(c) [**2 points**] The earth has mass 6.0×10^{24} kg, radius 6400 km and rotational period 24 hours. The mass accretion rate is 50×10^6 kg/year. Assuming a uniform mass distribution for earth, how many years must pass before the rotation period has changed by 1 second?

Useful moments of inertia are $\frac{2}{5}MR^2$ (uniform sphere) and $\frac{2}{3}mR^2$ (thin spherical shell of mass m, radius R).

C3. **(Takano)** A gas of 1 mole of fermions, whose energy ϵ is given by $\epsilon = c\hbar k$, occupies a volume V. Here c is a constant, and k the wave vector. Ignore the spin degree of freedom.

(a) [**3 points**] Express the Fermi energy, ϵ_F, of this gas in terms of V and Avogadro's number, N_A.

(b) [**2 points**] What is the energy of this gas at zero temperature?

(c) [**3 points**] Calculate the energy of this gas at low temperature, $T \ll \epsilon_F/k_B$, to the lowest order in T. Here k_B is Boltzmann's constant. Ignore the temperature dependence of the chemical potential. You may approximate the Fermi-Dirac distribution function as 1 for $\epsilon < \epsilon_F - k_B T$, 0 for $\epsilon > \epsilon_F + k_B T$, and $1/2$ in between.

(d) [**2 points**] Use the previous result to calculate the heat capacity of this gas to the lowest order in T.