Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, August 20, 2018, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

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A1. **(Thorn)** Consider a hydrogen atom immersed in a weak uniform electric field $\boldsymbol{E} = \mathcal{E}\hat{z}$. Treat the hydrogen atom as a proton and electron bound by the Coulomb potential, neglecting all relativistic and spin orbit corrections. You may disregard the electron spin (or fix it at $m_s = +1/2$ throughout this problem).

(a) [**3 points**] First recall the spectrum at zero field. Ignoring spin, give the degeneracies of the $n = 1, 2$ levels, and the angular momentum quantum numbers l, m_l and parity $\pm$ of every state belonging to each of these energy levels.

(b) [**2 points**] Explain, without detailed calculation, why the energy shift of the $n = 1$ level due to the electric field is quadratic in the field $\Delta E_{n=1} = -k\mathcal{E}^2$ for very weak field, and why k is positive.

(c) [**2 points**] Now consider the $n = 2$ energy level in light of its degeneracies. Explain why the weak electric field will split some of the degeneracy by an amount linear in the electric field.

(d) [**3 points**] Calculate the first order splittings of the $n = 2$ levels of hydrogen in a weak electric field. The zeroth order wave functions you might need are:

$$\psi_{210} = \sqrt{\frac{1}{32\pi a^5}} r \cos\theta e^{-r/2a}, \qquad \psi_{200} = \sqrt{\frac{1}{32\pi a^5}} (2a - r) e^{-r/2a}$$

where a is the Bohr radius.

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DEPARTMENT OF PHYSICS

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A2. **(Sikivie)** Consider a quantum mechanical system with a 2-dimensional Hilbert space. The Hamiltonian H and an observable Ω are given by the matrices

$$H = \begin{pmatrix} 7 & 4i \\ -4i & 13 \end{pmatrix} \text{eV}, \quad \Omega = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}$$

(a) [**3 points**] Find the eigenvalues and eigenstates of H.

(b) [**3 points**] Assume the system is in its lowest energy state at $t < 0$. At $t = 0$, Ω is measured. What are the possible outcomes of this measurement? What is the probability of each outcome?

(c) [**4 points**] Assume that the outcome of the measurement in part b. is the highest eigenvalue of Ω. Ω is measured again at a later time t. ($\hbar = 6.58 \times 10^{-16}$ eV s.) What are the possible outcomes of this second measurement and the corresponding probabilities?

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A3. **(Hirschfeld)** A particle, moving in one dimension, is in an infinite square well (of width a) has as its initial wave function an equal mixture of the first two stationary states:

$$\Psi(x,0) = C[\psi_1(x) + \psi_2(x)]$$

(a) **[2 points]** Find ψ_1, ψ_2, properly normalized, and the corresponding eigenvalues E_1 and E_2. Find C to normalise $\Psi(x,0)$.

(b) **[2 points]** Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express the latter in terms of sin and cos using $e^{i\theta} = \cos\theta + i\sin\theta$. Use $\omega = \pi^2\hbar/2ma^2$.

(c) **[2 points]** In terms of the wavefunction $\Psi(x,t)$, what is the definition of the expectation value $\langle\mathcal{O}\rangle(t)$ of a time-independent operator $\mathcal{O}(x)$ at time t?

(d) **[2 points]** Compute $\langle x\rangle$ and $\langle p\rangle$. Notice that they oscillate in time. What is the frequency of the oscillation? What is the amplitude?

(e) **[2 points]** Find the expectation value of the Hamiltonian operator, H, in terms of E_1 and E_2.