Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, August 15, 2019, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
   (e) Each problem is worth 10 points.
   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. **Relativistic Energy Correction for Harmonic Oscillator**. A particle of mass m moves in a one-dimensional harmonic oscillator potential with frequency ω. Allowing for relativistic effects, the kinetic energy can be written as

$$T = E - mc^2 = \sqrt{m^2c^4 + p^2c^2} - mc^2 \approx \frac{p^2}{2m} - \frac{p^4}{8m^3c^2}.$$

In this problem, we will treat the p^4 term as a perturbation.

(a) [**2 points**] Write down the *unperturbed* Hamiltonian H_0 in terms of the P and X operators.

(b) [**2 points**] Write down the *unperturbed* Hamiltonian H_0 in terms of the ladder (raising and lowering) operators, $a_+,\ a_-$.

(c) [**2 points**] Give the formula for the *first order energy shift, E_n^1*, of the eigenstates $|n\rangle\,,\ n = 0, 1, 2, 3......$ of the unperturbed Hamiltonian in terms of the perturbing Hamiltonian using $a_+,\ a_-$. Note that you do not need to evaluate the matrix elements.

(d) [**4 points**] Now, for the ground state, $|0\rangle$, explicitly evaluate the matrix element to get the 1^{st} order relativistic shift to the ground state energy of the harmonic oscillator.

Useful Formulae:

$$a_+ = \sqrt{\frac{m\omega}{2\hbar}}\, X - i\frac{1}{\sqrt{2\hbar m\omega}}\, P; \quad a_- = \sqrt{\frac{m\omega}{2\hbar}}\, X + i\frac{1}{\sqrt{2\hbar m\omega}}\, P.$$

$$a_+ |n\rangle = \sqrt{n+1}\, |n+1\rangle\,; \qquad a_- |n\rangle = \sqrt{n}\, |n-1\rangle\,.$$

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A2. Consider a particle in an Infinite square well potential between $x = 0$ and $x = L$, $(0 < x < L)$. The potential $V = \infty$ at $x = 0$ and at $x = L$; $V = 0$ for $0 < x < L$. In the second excited state, $n = 3$, the wave function of the particle $\Psi_3 = A_3 sin 3\pi x/L$.

(a) [**2 points**] In order that the probability of finding the particle in the box equals 1, what is A_3?

(b) [**2 points**] What is the ratio between the energy of the particle in the $n = 3$ state to the energy of the particle in the $n = 1$ state?

(c) [**1 point**] Is the wavefunction Ψ_3 symmetric or antisymmetric around $x = L/2$?

(d) [**5 points**] Calculate the probability of finding the particle between $x = L/3$ and $x = L/2$.

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A3. **Scattering in one dimension:** A particle of mass m, moving in one dimension, is incident from the left on a delta function potential

$$V(x) = \lambda\delta(x) \tag{1}$$

with momentum $p = k\hbar$. In this problem you are to obtain the probabilities that the particle is reflected and transmitted. The delta function potential imposes the following matching conditions on the wave function and its derivative at $x = 0$

$$\psi_<(0) = \psi_>(0) = \psi(0) \tag{2}$$

$$\frac{d\psi_>}{dx}(0) - \frac{d\psi_<}{dx}(0) = \frac{2m\lambda}{\hbar^2}\psi(0). \tag{3}$$

Here $\psi_>(x) = \psi(x)$ for $x > 0$ and $\psi_<(x) = \psi(x)$ for $x < 0$. We see that the wave function is continuous at $x = 0$ but its derivative has a discontinuity there (if $\psi(0) \neq 0$). You may assume these matching conditions without derivation in the following, but if you can derive them in part e), you will receive 1 (and only 1) point.

(a) [**2 points**] Write down the plane wave forms of $\psi_<(x)$ and $\psi_>(x)$ describing this reflection/transmission process. Call r the amplitude for reflection and call t the amplitude for transmission.

(b) [**2 points**] Use the matching conditions stated above to obtain a pair of linear equations in r and t.

(c) [**3 points**] Solve the equations of part b) for r and t, and determine the *probabilities* of reflection and transmission

(d) [**2 points**] Discuss the low and high energy behaviors of these probabilities.

(e) [**1 point**] Prove the matching conditions stated at the beginning of the problem.