Student ID Number: _________

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part B, 15 August 2002, 14:00 - 17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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B1. Suppose the time-dependent Hamiltonian $\mathcal{H}$ of a one-dimensional quantum mechanical system is given by

$$\mathcal{H}(x,t) = \mathcal{H}_0(x) + \epsilon V(x,t) \quad ,$$

where $\mathcal{H}_0$ is the static Hamiltonian and $\epsilon V(x,t)$ is a perturbing potential.

(a) (5 points) Derive the most general solution to the Schrödinger equation for the time-dependent Hamiltonian to first order in ϵ. Assume that $\phi_i(x)$ and E_i are the eigenfunctions and eigen-energies corresponding to $\mathcal{H}_0$ and that the system is in a pure state at $t = 0$.

(b) (5 points) A simple two level quantum mechanical system possessing an electric dipole has energies E_1 and E_2. Suppose this system is driven by a time-dependent electric field of the form

$$E(t) = A_0 \cos(ft) \quad .$$

The perturbing potential can then be written in the form

$$\epsilon V(x,t) = -e\,E(t)\,x \quad ,$$

where e is the charge of an electron. If the system is initially in the ground state at $t = 0$, find the probability that the system will be in the upper state at time t for frequencies near the resonant frequency $(E_2 - E_1)/\hbar$. Justify any assumptions you may make.

B2. This problem involves aspects of a solenoid.

(a) (3 points) Calculate the self-inductance L per unit length of an infinite solenoid with N turns per unit length and radius R.

(b) (5 points) Suppose the current, I, in the solenoid varies linearly with time, such that

$$I(t) = \alpha t + \beta \quad ,$$

where α and β are constants. Assume that this time dependence for the current was established a long time ago. (In other words, any short lived transient effects that may be present when the current first starts flowing may be neglected.) Find expressions for the induced $\vec{E}$ field both inside and outside the solenoid.

(c) (2 points) In magnetostatics, you learned that the $\vec{B}$ field is uniform inside an infinite solenoid and it is zero everywhere outside. Do you think that this is true for electrodynamics? Without explicitly calculating the induced $\vec{B}$ field, (continued on the next page)

justify your answer by considering what would happen if the current in the solenoid additionally had a quadratic dependence on time, *i.e.*

$$I(t) = \gamma t^2 + \alpha t + \beta \quad ,$$

where γ is a constant.

B3. A homogeneous billiard ball of mass m and radius R moves on a horizontal table. The coefficient of kinetic friction between the table and the ball is μ. The ball is struck by a cue, which delivers an impulsive force **F** of duration Δt; the point of impact is a height h above the table, and the cue makes an angle θ with respect to the horizontal.

(a) (2 points) What is the moment of inertia of the ball about its center of mass?

(b) (3 points) Assume that $\theta = 0$. What is the value of h required so that the cue ball rolls without slipping immediately after being struck? You should be able to express your result in terms of R.

(c) (5 points) Show that it is possible to have the ball return after being struck if

$$\tan\theta \geq \frac{1}{\sqrt{2R/h - 1}} \quad .$$