PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part D, 13 August 2004, 14:00 - 17:00

D1. A linear array of N identical coherent oscillators emitting radiation with wavelength λ are spaced a distance d apart as shown in the figure. The rays shown are almost parallel and meet at a point P that is located on the distant plane Σ at an angle θ with respect to the central axis (given in the figure as the horizontal dashed line) of the array.

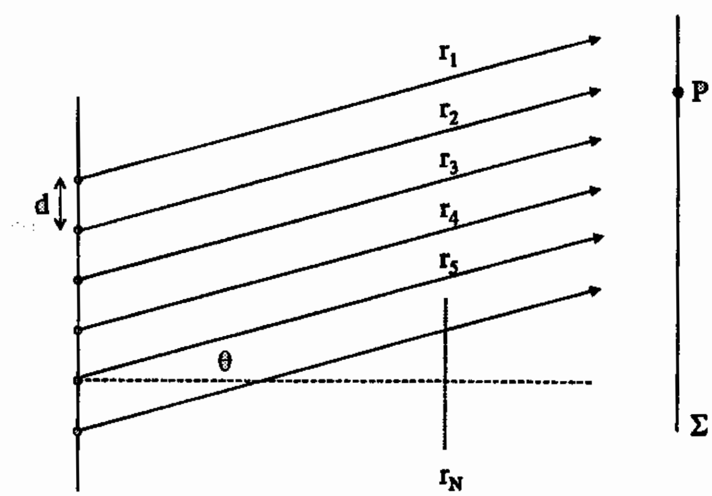
(a) (4 points) Find an expression for the sum of the interfering wavelets at P from each of the oscillators where the electric field amplitude from the j^{th} oscillator at P can be written as

$$E_j(r_j) = E_o(r)\,\exp[\,i(\,k\,r_j - \omega\,t\,)\,] \quad .$$

Since P is far away, you can ignore any variations in the prefactor $E_o(r)$ and need only consider the optical path length differences that appear in the exponents.

(b) (2 points) Using the result found above, calculate the total intensity at P in terms of the parameters λ, θ, d, N, and I_o, where I_o is the intensity at P from any single source.

(c) (4 points) The central maximum of the beam can be steered by electronically introducing a phase shift ϵ between adjacent oscillators. For d = 7 m, $\lambda = 21$ cm, and $N = 32$, find the minimum value of ϵ in degrees which centers a maximum in the total intensity pattern at a point located at $\theta = 30°$.



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D2. A macroscopic number, N, of non-interacting Bose particles with mass m and spin $S = 0$ are confined in an anisotropic harmonic potential, V, where

$$V(r) \;=\; \frac{1}{2} m \left(\omega_1^2 x^2 \;+\; \omega_2^2 y^2 \;+\; \omega_3^2 z^2\right) \quad .$$

(a) (4 points) Show that the total number of states with energy less than ϵ may be written as

$$G(\epsilon) \;=\; \frac{\epsilon^3}{6\,\hbar^3\,\omega_1\,\omega_2\,\omega_3} \quad .$$

Ignore the zero point energy since N is large and assume $\hbar\omega_i \ll k_B T$, where k_B is the Boltzmann constant.

(b) (3 points) The Bose-Einstein condensation temperature T_o is defined as the highest temperature at which a macroscopic number of particles occupy the lowest energy state and the chemical potential is zero. Show

$$k_B T_o \;=\; \left(\frac{2\,\hbar^3\,\omega_1\,\omega_2\,\omega_3\,N}{\Gamma(3)\,\zeta(3)}\right)^{1/3} \quad .$$

Here, $\Gamma(\alpha)$ is the gamma function, $\Gamma(\alpha) \;=\; \int_0^\infty x^{\alpha-1}\,e^{-x}\,dx$, and $\zeta(\alpha)$ is the Riemann zeta function, $\zeta(\alpha) \;=\; \sum_{\ell=1}^\infty \ell^{-\alpha}$.

(c) (3 points) The condensate fraction, $N_o(T)$, below T_o has the following temperature dependence,

$$N_o(T) \;=\; N\left\{1 - \left(\frac{T}{T_o}\right)^n\right\} \quad .$$

What is the value of n?

D3. At the Earth's equator there are often long periods of time when the winds disappear, trapping sailing vessels for days or weeks (equatorial doldrums).

Suppose that a small sailboat is becalmed at the equator. The captain, a former physics major, decides to put the boat into motion by raising the anchor ($m = 200$ kg) to the top of the mast ($h = 20$ m). The rest of the boat has a mass $M = 1000$ kg. In case you need the value of the Earth's radius, you may take it to be $R_E = 6400$ km.

(a) (2 points) Why will the boat begin to move?

(b) (2 points) In which direction will the boat move?

(c) (6 points) What is the boat's final velocity (with respect to the Earth)?