Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida

Part A, 18 August 2005, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. A particle of mass m moves in one dimension inside a box, with walls at $x = \pm a$. Inside, there is an attractive δ-function potential at $x = 0$. The complete potential $V(x)$ is given by

$$\frac{2m}{\hbar^2} V(x) \; = \; -\lambda\,\delta(x) \quad \text{for} \quad |x| < a$$

and

$$V(x) \; = \; \infty \quad \text{at} \quad x \; = \; \pm a \quad .$$

Consider the energy eigenvalue problem for negative energies $E < 0$.

(a) (2 points) The δ-function potential requires that the wavefunction $\psi(x)$ is continuous at $x = 0$ but its derivative is not. Find the discontinuity of the derivative of ψ at $x = 0$ by integrating the second derivative of ψ from $-\varepsilon$ to $+\varepsilon$ where ε is infinitesimal.

(b) (4 points) Set up the equation which determines the allowed values of E, if any.

(c) (2 points) Under what conditions on λ and a are there more than one negative energy solutions?

(d) (2 points) Under what conditions on λ and a are there no negative energy solutions?

A2. A plane wave, $E_\circ \exp i(\vec{k}\cdot\vec{r} - \omega\, t)$, with intensity $I_\circ \propto E_\circ^2$ from a coherent source is incident on a semi-infinite slit of width b. According to the Huygens-Fresnel Principle, every unobstructed point on the aperture slit serves as the source of cylindrical secondary wavelets with the same wavelength λ as the incident wave. The amplitude of the optical disturbance (electric field) at point P on a screen, a distance s away, is the superposition (magnitude and phase) of all these wavelets. In the following, ignore the inverse square root dependence of the cylindrical wave amplitude on distance and use the paraxial approximation where θ is small so that

$$\sin(\theta) \sim \theta \sim \frac{y}{s} \quad .$$

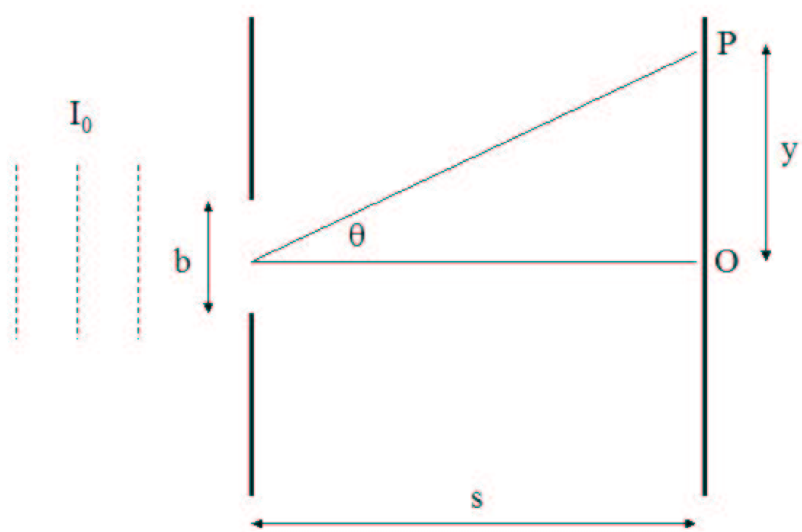
In these approximations, the magnitude of each secondary wavelet incident on the screen is the same but the phase can vary.

(a) (3 points) Find the total electric field $E_{\rm P}$ at point P. (Hint: A convenient way to do this is to break up the slit into N smaller slits, each of width (b/N), and recognize that there is a constant phase factor between successive terms. Calculate the sum and then let N approach infinity.)

(b) (3 points) Find the intensity distribution I(y) on the screen. Your answer should depend on $I_\circ$, y, b, θ, λ and s.

(c) (2 points) Show quantitatively that when one doubles the width b, the intensity on the central axis at O increases by a factor of 4.

(d) (2 points) Reconcile your answer in (c) with the fact that the energy passing through the slit only doubles.



A3. Solid ice at temperature $T = -10^\circ$ C exhibits an equilibrium vapor pressure $p_0 = 260$ Pa. It is an acceptable approximation to treat the water vapor at this low temperature and pressure as an ideal gas.

(a) (6 points) Suppose that an ice cube (1 cm^3) is placed inside a small plastic box. The box is stored in a kitchen freezer at $T = -10^\circ$ C. The box is not completely airtight. It has a tiny circular hole (diameter $d = 100$ μm) that allows water vapor to escape. Estimate how long will it take until the ice cube has completely vanished. Give your answer in days.

You should assume that the freezer is "frostless", meaning that it efficiently removes free water vapor from its interior space.

(b) (4 points) Now suppose there is no plastic box. That is, imagine a spherical ball of ice (radius = 10 cm) placed into a vacuum chamber where the walls are maintained at $T = -20^\circ$ C. At this temperature, the equilibrium vapor pressure of ice is $p_0 \approx 100$ Pa. Estimate how long it will take before the ice ball has completely evaporated. State clearly any important assumptions that you have made.

You might find the following facts to be useful:
$k_B = 1.3807 \times 10^{-23}$ J/(mol K)
$N_{\text{Avogadro}} \approx 6.0221 \times 10^{23}$ 1/mol
H_2O molecular weight $\approx$ 18.0 g/mol
Density of ice = 0.92 g/cm^3