

Student ID Number: _____

PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part B, 18 August 2005, 14:00 - 17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *“On my honor, I have neither given nor received unauthorized aid in doing this assignment.”*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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- B1. Two different types of states are often used to describe laser fields: The number state which is the eigenstate of the Hamiltonian of the harmonic oscillator $|n\rangle$ and the coherent state which is the eigenstate of the lowering operator $|\alpha\rangle$:

$$H|n\rangle = \hbar\omega \left(\hat{a}_+ \hat{a}_- + \frac{1}{2} \right) |n\rangle \quad \text{and} \quad \hat{a}|\alpha\rangle = \alpha|\alpha\rangle .$$

Recall also the following properties:

$$\hat{a}_-|n\rangle = \sqrt{n}|n-1\rangle , \quad \hat{a}_+|n\rangle = \sqrt{n+1}|n+1\rangle , \quad \text{and} \quad |\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle .$$

The phase operator is defined by the relations

$$e^{\hat{x}p}(-i\phi) = \hat{a}_+ (\hat{n} + 1)^{-\frac{1}{2}} \quad \text{and} \quad e^{\hat{x}p}(i\phi) = (\hat{n} + 1)^{-\frac{1}{2}} \hat{a}_- , \quad \text{where} \quad \hat{n} \equiv \hat{a}_+ \hat{a}_- .$$

- (a) (3 points) Show that

$$\begin{aligned} e^{\hat{x}p}(-i\phi)|n\rangle &= |n+1\rangle \quad \text{for} \quad n \geq 0 , \\ e^{\hat{x}p}(i\phi)|n\rangle &= |n-1\rangle \quad \text{for} \quad n > 0 , \quad \text{and} \\ e^{\hat{x}p}(i\phi)|n\rangle &= 0 \quad \text{for} \quad n = 0 . \end{aligned}$$

- (b) (3 points) $e^{\hat{x}p}(i\phi)$ is not a Hermitian operator and does not represent an observable property. However, it can be used to produce another pair of operators:

$$\hat{c}\phi \equiv \frac{1}{2} \{ e^{\hat{x}p}(i\phi) + e^{\hat{x}p}(-i\phi) \} \quad \text{and} \quad \hat{s}\phi \equiv \frac{1}{2i} \{ e^{\hat{x}p}(i\phi) - e^{\hat{x}p}(-i\phi) \} .$$

Calculate the standard deviation of these operators for the number states (pay attention to $|n=0\rangle$).

- (c) (4 points) Calculate the expectation value $\langle \alpha | \hat{c}\phi | \alpha \rangle$ for $|\alpha|^2 \gg 1$. Use the asymptotic expansions:

$$\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n! \sqrt{n+1}} = \frac{e^{|\alpha|^2}}{|\alpha|} \left(1 - \frac{1}{8|\alpha|^2} + \dots \right), \quad |\alpha|^2 \gg 1 .$$

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- B2. The two halves of a conducting sphere are separated by a thin insulating gap in the equatorial plane. The top and bottom hemispheres are held at constant potentials $\pm V_0$.
- (a) (5 points) What is the leading term in the potential far from the sphere?
 - (b) (5 points) The potential applied to the two halves of the sphere now oscillates slowly, $\pm V_0 \cos \omega t$. What is the power radiated?
- B3. (10 points) A mouse of mass $m = 200$ g runs radially outward on a merry-go-round, which is turning at a constant angular speed of 10 rpm (*i.e.* revolutions per minute). The speed of the mouse is constant at 0.5 m/s relative to the merry-go-round. In the rotating frame, the mouse moves at constant speed in a straight line (*i.e.* unaccelerated motion). Find the magnitude (in Newtons) and direction (relative to the radially inward direction) of the force of friction that the surface of the merry-go-round exerts on the mouse when it is 2 m from the axis of rotation.