Student ID Number: ___________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, 17 August 2006, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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B1. Consider a particle of mass m moving in a one dimensional potential $V(x)$

$$V(x) = \begin{cases} +\infty & \text{for } x \leq -a \\ \lambda\delta(x) & \text{for } -a < x < a \\ +\infty & \text{for } x \geq a \end{cases}$$

In other words the particle is confined to a one-dimensional box of length $2a$ with walls at $x = \pm a$, and there is a repulsive delta function barrier ($\lambda \geq 0$) at the center of the box. The physics of this potential thus resembles that of a double well.

(a) (3 points) First find all the even parity and odd parity energy eigenvalues and eigenfunctions for the case of the simple box with $\lambda = 0$. Treat the two parities separately.

(b) (1 point) In the following parts we consider $\lambda > 0$. The effect of the delta function barrier is to impose the following matching conditions on the wave function and its derivative at $x = 0$:

$$\psi_<(0) = \psi_>(0) = \psi(0)$$

$$\frac{d\psi_>}{dx}(0) - \frac{d\psi_<}{dx}(0) = \frac{2m\lambda}{\hbar^2}\psi(0)$$

Here $\psi_>(x) = \psi(x)$ for $x > 0$ and $\psi_<(x) = \psi(x)$ for $x < 0$. We see that the wave function is continuous at $x = 0$ but its derivative has a discontinuity there (if $\psi(0) \neq 0$). Note however that, since odd parity wave functions vanish at $x = 0$, their derivatives are continuous at $x = 0$. You may assume these matching conditions without derivation in the following, but if you can derive them here, you will receive 1 (and only 1) point.

(c) (3 points) Making use of the matching conditions stated in (b) obtain the transcendental equation (it involves trig functions) determining the even parity energy eigenvalues for $\lambda > 0$. Indicate the solution graphically by plotting the two sides of this equation.

(d) (1 point) What happens to the odd parity levels when $\lambda > 0$?

(e) (2 points) Explain what happens to the energy spectrum when $\lambda \to +\infty$, by giving all of the energy levels and the degeneracy of each in that limit.

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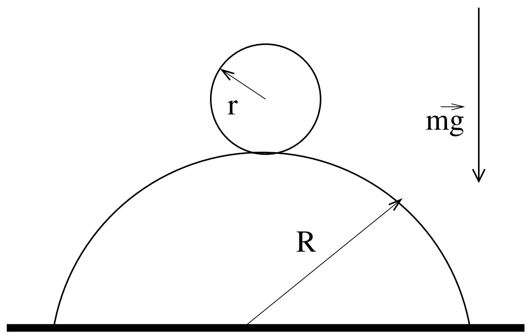
B2. A uniform hoop of mass m and radius r starts to roll without slipping from the top of a fixed semi-cylinder of radius R, as shown in the figure below.



(a) (3 points) Write down the Lagrangian and the constraints for the system in suitable coordinates.

(b) (3 points) Use Lagrange's equations to find the point at which the hoop falls off the semi-cylinder.

(c) (4 points) Alternatively, use energy conservation and Newton's Second Law to find the point at which the hoop falls off the semi-cylinder.

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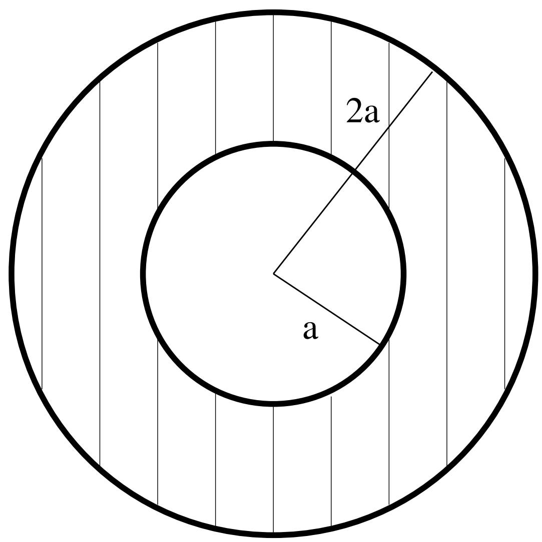
B3. Two concentric conducting spherical shells, with radii a and $2a$, have charge $+Q$ and $-Q$ respectively. The space between the shells is filled with a linear dielectric with permittivity

$$\varepsilon(r) = \frac{\varepsilon_0 a}{1.5a - 0.5r} \tag{1}$$

which varies with radial distance r from ε_0 at $r = a$ to $2\varepsilon_0$ at $r = 2a$. Here ε_0 is the dielectric constant of vacuum.



(a) (1 point) Use Gauss's Law to determine the displacement field $\mathbf{D}$ between the spherical shells.

(b) (3 points) Determine *all the bound* charge density (both bulk density ρ_B and surface density σ_B) between the spherical shells. (Hint: You know $\mathbf{D}$ from above, so you can then determine the electric field $\mathbf{E}$ for a linear dielectric and hence you can then determine the local polarization $\mathbf{P}$).

(c) (1 point) Verify explicitly that the *total* charge of the dielectric (both on the surface and throughout the volume) is zero.

(d) (3 points) Determine the total energy U of this system, from the relation $U = \frac{1}{2}\int \mathbf{D} \cdot \mathbf{E}\, d\tau$. Or alternatively you can calculate the potential difference V between the charged spheres.

(e) (2 points) Determine the capacitance of this system.