Student ID Number: __________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, 18 August 2006, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. Consider a hydrogen atom (described in the absence of external fields by the Hamiltonian H_0) immersed in a magnetic field $\overrightarrow{B} = B\hat{z}$ defined to lie along the z-axis.

(a) (1 point) What additional term H_1 must be added to H_0 to describe the interaction of the the electron with the magnetic field (the total magnetic dipole moment is typically referred to as $\overrightarrow{\mu}$) ?

(b) (2 points) For sufficiently large B fields the orbital and spin angular momenta can be considered to be independent of each other. What is H_1 in terms of $\overrightarrow{L}$, $\overrightarrow{S}$, $\overrightarrow{B}$, and μ_b (the Bohr magneton), $g_l = 1$, and $g_s = 2$?

(c) (3 points) What is the energy shift of a level with quantum numbers m_l, and m_s (with respect to the zero field energies)? What is the energy splitting between the spin up/down states for a given value of m_l as a function of the magnetic field strength B ?

(d) (4 points) If the magnetic field B is sufficiently weak ($B \leq 1\,\mathrm{Tesla}$) then spin orbit coupling cannot be ignored. In this case the natural states to work with are the total angular momentum eigenstates $\overrightarrow{J} = \overrightarrow{L} + \overrightarrow{S}$, and the magnetic field interaction can be treated as a perturbation term. What is the first order energy shift for an energy level with $j = l \pm \frac{1}{2}, m_j$?

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C2. A particle of mass m moves along the x axis in a potential energy $U(x) = \frac{1}{2}m\omega^2 x^2$ where ω is a constant. The particle is in contact with a heat reservoir at temperature T.

(a) (4 points) Suppose that the system is at extremely low temperatures, where a quantum mechanical description is appropriate. Find the average energy as a function of T.

(b) (3 points) At higher temperature a classical description is acceptable. Show that your result for part (a) reduces to the "correct" classical result for a harmonic oscillator.

(c) (3 points) Suppose that the potential energy now includes a $weak$ anharmonic term

$$U(x) = \frac{1}{2}m\omega^2 x^2 + \lambda x^3.$$

Assume that the system is in the classical limit but the temperature is low enough that

$$\lambda \ll \frac{(m\omega^2)^{3/2}}{(k_B T)^{1/2}}$$

Find an expression for the average position $\langle x \rangle$ as a function of T, keeping only terms up to linear in λ. You may find it useful to approximate some of the integral expressions (neglecting terms of order λ^2 and higher) in order to evaluate them.

Potentially useful facts:

(1) If $|x| < 1$ then $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

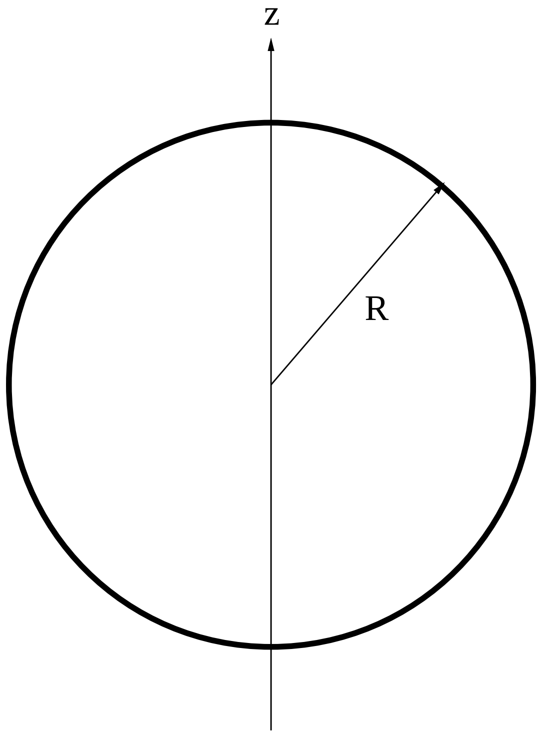
(2) $\int_{-\infty}^{\infty} dx\ e^{-x^2} = \sqrt{\pi}$

(3) $\int_{-\infty}^{\infty} dx\ x^2 e^{-x^2} = \sqrt{\pi}/2$

(4) $\int_{-\infty}^{\infty} dx\ x^4 e^{-x^2} = 3\sqrt{\pi}/4$

(5) $\int_{-\infty}^{\infty} dx\ x^6 e^{-x^2} = 15\sqrt{\pi}/8$

C3. A uniformly charged solid spherical shell of radius R carries total surface charge Q, and is set spinning with angular velocity ω about the z axis passing through its center as shown in figure.



(a) (3 points) What is the magnetic dipole moment of the shell?

(b) (2 points) Find the average magnetic field inside the shell.

(c) (2 points) Find the approximate vector potential at a point (r, θ) where $r >> R$.

(d) (3 points) Find the magnetic field at the center of the shell, and check that it is consistent with part (b).