Student ID Number: ___________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, 22 Aug 2008, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. Consider two quantum harmonic oscillators, represented by the ladder operators which satisfy the canonical commutation relations

$$[\, a\, ,\, a^\dagger\,] \;=\; [\, b\, ,\, b^\dagger\,] \;=\; 1\;,$$

all other commutators being zero. The Hilbert space is constructed from the state $|\,0\rangle$ which satisfies

$$a\,|\,0\rangle \;=\; b\,|\,0\rangle \;=\; 0\;, \qquad \langle 0\,|\,0\rangle \;=\; 1\;.$$

The Hamiltonian is given by

$$H_0 \;=\; \hbar\omega\left(a^\dagger a + b^\dagger b + 1\right)\;.$$

(a) (*2 points*) Enumerate and construct the normalized H_0 eigenstates with energy $3\hbar\omega$.

(b) (*2 points*) Consider the new operator

$$A \;=\; \alpha\, a^\dagger \;+\; \beta\, b\;,$$

and its hermitian conjugate $A^\dagger$, where α and β are real. Find α and β such that A and $A^\dagger$ are themselves canonical ladder operators.

(c) (*2 points*) Derive the expression for a second canonical set of ladder operators, B and $B^\dagger$ which commute with A and $A^\dagger$.

(d) (*2 points*) In the limit $\alpha << \beta$, construct the new ground state $|\,\Omega\rangle$ which satisfies

$$A\,|\,\Omega\rangle \;=\; B\,|\,\Omega\rangle \;=\; 0\;.$$

(e) (*2 points*) The two oscillators are now weakly coupled, resulting in the Hamiltonian

$$H \approx\; H_0 + \hbar\omega\lambda(a^\dagger b^\dagger + a\, b\,)\;,$$

where λ is a *very small* dimensionless parameter. Rewrite H in terms of A, B and their conjugates to determine the energy of its ground state.

C2. (*10 points*) Find the work required to compress 22.4 liters of some gas, such as Oxygen or Nitrogen or Argon, initially at atmospheric pressure (1 Atm = 10^5 N/m^2), to a volume of 11.2 liters. The process is done isothermally at room temperature. (If unable to compute the exact solution, as a last resort, make an order of magnitude estimate.)

PRELIMINARY EXAMINATION
Department of Physics
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Part C, Aug 21 2008, 09:00–12:00

C3. A spherical shell with a radius R has a surface charge density given by:

$$\sigma(\theta) = k\cos^2\theta$$

where k is a constant, and θ is the polar angle.

(a) (*2 points*) Write down all the relevant boundary conditions for the electric potential.

(b) (*1 point*) What is the expression for $\sigma(\theta)$ in terms of Legendre polynomials?

(c) (*5 points*) Find the potential inside and outside the shell.

(d) (*2 points*) Compare the result in (c) to the multipole expansion of the electric potential of the shell at large distances. Based on this, find the electric dipole moment of the shell. The general expression for the multipole expansion of the potential of a volume charge distribution $\rho(\vec{r})$ is

$$V(r,\theta) = \frac{1}{4\pi\epsilon}\sum_{0}^{\infty}\frac{1}{r^{n+1}}\int r' P_n(\cos\theta)\rho(\vec{r'})d\tau'$$

where τ is a small volume element.

Useful Formulas:

$$P_0(x) = 1 \tag{1}$$
$$P_1(x) = x \tag{2}$$
$$P_2(x) = (3x^2 - 1)/2 \tag{3}$$
$$P_3(x) = (5x^3 - 3x)/2 \tag{4}$$
$$P_4(x) = (35x^4 - 30x^2 + 3)/8 \tag{5}$$
$$P_5(x) = (63x^5 - 70x^3 + 15x)/8 \tag{6}$$