Student	ID Number:	
Sundent	HIJ NUHHDEL	

DEPARTMENT OF PHYSICS UNIVERSITY OF FLORIDA Part D, 22 Aug 2008, 14:00–17:00

# Instructions

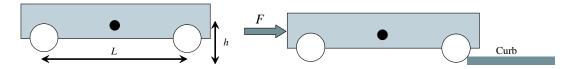
- 1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may NOT use programmable calculators to store formulae.
- 2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
- 3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
- 4. Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.
- 5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
- 6. Each problem is worth 10 points.
- 7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."

### DO NOT OPEN EXAM UNTIL INSTRUCTED

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D1. A car of mass M has a distance L between the front and back wheels, a distance w between the left and right wheels, and the center of mass located a distance h above the ground. All wheels have radius  $r_w$ . You can assume for this problem that the car has rear wheel drive, is



right-left and front-rear symmetric and that its weight is supported equally by the front and rear wheels.

- (a) (4 points) If the car accelerates from rest, what is the maximum acceleration a that the car can sustain before the front wheels leave the ground? Express a in terms of the acceleration of gravity g and the other parameters of the problem. (Assume a very large coefficient of static friction between the wheels and the ground.)
- (b) (2 points) The car is at rest with its front wheels against a rectangular shaped curb of height  $h_c \ll r_w$ . What minimum force F applied from behind the car is necessary to push the front wheels over the curb?
- (c) (2 points) The car now moves around a turn with radius of curvature R. What is the maximum velocity that the car can round the turn without the inside wheels leaving the ground? Assume that the turn is level, and the coefficient of friction is large.
- (d) (2 points) The tendency for a car to roll over when rounding a curve at high speed can be counterracted by mounting a large spinning flywheel to the car's frame. Draw a figure showing the direction of the car and the direction of the angular momentum of the flywheel that would cause this to happen.
  - Show that for a disk-shaped flywheel of mass m and radius r that the requirement for equal weight on the inside and outside wheels is that the angular velocity  $\omega$  of the flywheel is related to the velocity of the car v by  $\omega = 2vMh/mr^2$ . Assume that the turn is level.

Note: The moment of inertia for a uniform disk is  $I = \frac{1}{2}mr^2$ .

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- D2. A simple solid consists of molecules whose centers of mass are fixed at the lattice sites of a close packed lattice and each molecule having an electric dipole moment that interacts with its nearest neighbors.
  - (a) (3 points) If the mean polarization of each molecule is P, show that the configurational entropy per molecule is

$$S = \ln 2 - \frac{1}{2}(1+P)\ln(1+P) - \frac{1}{2}(1-P)\ln(1-P)$$

so that  $S = \ln 2$  for P = 0 and S = 0 for  $P = \pm 1$ .

- (b) (4 points) If the interaction energy is  $E=-AP^2$ , show that, depending on the sign of A, there can be a transition from a disordered state (P=0) to an ordered state ( $P\neq 0$ ). Calculate the transition temperature,  $T_c$ .
- (c) (3 points) Sketch the free energy functional as a function of P for (i)  $T > T_c$ , (ii)  $T = T_c$ , (iii)  $T < T_c$ .

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D3. The complex frequency-dependent dielectric constant  $\tilde{\epsilon}(\omega)$  of a medium can be written in the form

$$\tilde{\epsilon}(\omega) = \epsilon_1(\omega) - i\epsilon_2(\omega) \tag{1}$$

where  $\epsilon_1(\omega)$  represents the polarization response and  $\epsilon_2(\omega)$  represents the dissipation (loss) at finite frequencies ( $\epsilon_1$  and  $\epsilon_2$  are real). If this medium is used as the dielectric in a parallel plate capacitor with electrodes having an area A and separation d the capacitance becomes complex and can be written as

$$\tilde{C}(\omega) = \epsilon_0 \tilde{\epsilon}(\omega) A/d = C_1(\omega) - iC_2(\omega)$$
(2)

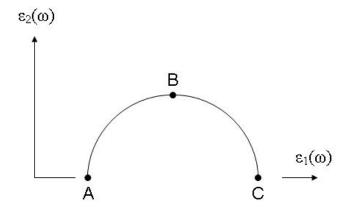
with the real and imaginary parts again respectively representing polarization and loss. Note that  $\tilde{\epsilon}(\omega)$  is a dimensionless quantity normalized to the vacuum permittivity  $\epsilon_o$ . The minus signs in equations (1) and (2) are chosen so that that the quantities  $\epsilon_2(\omega)$  and  $C_2(\omega)$ , which represent dissipation, are positive.

- (a) (1 point) Calculate the complex impedance  $\tilde{Z}(\omega)$  of a frequency-dependent resistance  $R(\omega)$  in parallel with  $C_1(\omega)$  and find a relationship between the loss components  $R(\omega)$  and  $C_2(\omega)$ .
- (b) (2 points) The Debye response for dipoles having a single relaxation time  $\tau$  can be written in the form

$$\tilde{\epsilon}(\omega) = \epsilon(\infty) + \frac{\epsilon(0) - \epsilon(\infty)}{1 + i\omega\tau}$$

where  $\epsilon(0)$  and  $\epsilon(\infty)$  are real and represent the zero and infinite frequency responses respectively. Noting that  $\epsilon(0) - \epsilon(\infty) \geq 0$ , calculate and then sketch both  $\epsilon_1(\omega)$  and  $\epsilon_2(\omega)$  as a function of  $\omega$ .

Cole-Cole plots can be constructed for any complex response function by using  $\omega$  as an implicit variable and plotting the imaginary part of the response versus the real part. For the Debye response the Cole-Cole trajectory is a semicircle as shown schematically in the figure next page.



- (c) (2 points) In this Cole-Cole space, specify values of  $\epsilon_1(\omega)$  and  $\epsilon_2(\omega)$  at points A, B and C shown in the figure. In addition, show with an arrow the direction for increasing frequency.
- (d) (3 points) Show explicitly using your answers to Part (b) that the Cole-Cole trajectory for all  $\omega$  is a semicircle.
- (e) (2 points) Calculate explicitly the zero and infinite frequency values of  $C_1(\omega)$  and  $R(\omega)$ . Your answers in conjunction with the parallel resistance model of Part A should convince you that the dissipation for this lossy capacitor is zero at these limits.