Prelim ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, 09:00–12:00, Aug 19, 2010

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work **different problems on separate sheets of paper**. The sheets for each problem will be stapled together but separately from the other two problems.
4. You will be assigned a **Prelim ID Number**, *different from your UF ID Number*. The **Prelim ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of **each sheet**. Do **NOT** use your name or UF ID Number anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. A non-conducting sphere of radius R caries a charge density

$$\rho = \frac{k}{r}$$

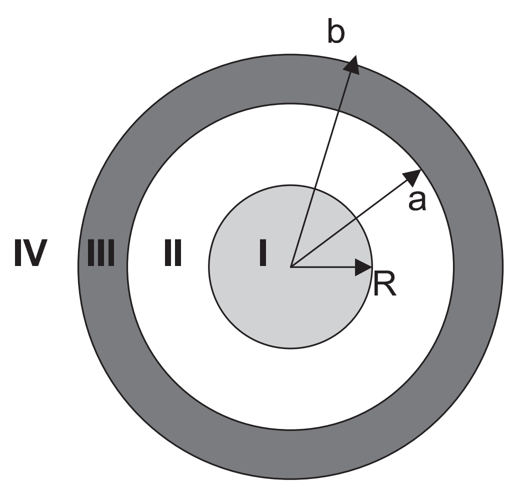
in the region $r \leq R$, where k is a positive constant and r is the distance from the center of the sphere (see the inclosed figure). The sphere is surrounded by a thick, concentric conducting metal shell with an inner radius a and an outer radius b. The shell carries no net charge.

(a) (*2 points*) Find the surface charge density at the inner and the outer surface of the conductor.

(b) (*4 points*) Find the electric field $\vec{E}$ in all four regions (i) $r < R$; (ii) $R < r < a$, (iii) $a < r < b$, and (iv)$r > b$

(c) (*3 points*) Find the potential V at the center of the sphere using infinity ($r = \infty$) as a reference point.

(d) (*1 point*) If the outer shell is grounded, what would be the potential at the center of the sphere using the same reference point as in part c?



A2. Consider a quantum mechanical system with a 2-dimensional Hilbert space. In the basis formed by the energy eigenstates, the Hamiltonian H and an observable Ω are given by the matrices ($i = \sqrt{-1}$)

$$H = \begin{pmatrix} 7 & 0 \\ 0 & 11 \end{pmatrix} \text{eV}, \quad \Omega = \begin{pmatrix} 13 & -i3 \\ i3 & 5 \end{pmatrix} .$$

(a) (*2 points*) Find the eigenvalues and eigenstates of Ω.

(b) (*3 points*) Assume the system is in its lowest energy state at $t < 0$. At $t = 0$, the observable Ω is measured. What are the possible outcomes of this measurement? What is the probability of each outcome?

(c) (*5 points*) Assume that the outcome of the measurement in part b is the lowest eigenvalue of Ω. At $t = 2.3\ 10^{-15}$ sec, Ω is measured again. What are the possible outcomes of this second measurement and the corresponding probabilities? ($\hbar = 6.63\ 10^{-16}$ eV·sec).

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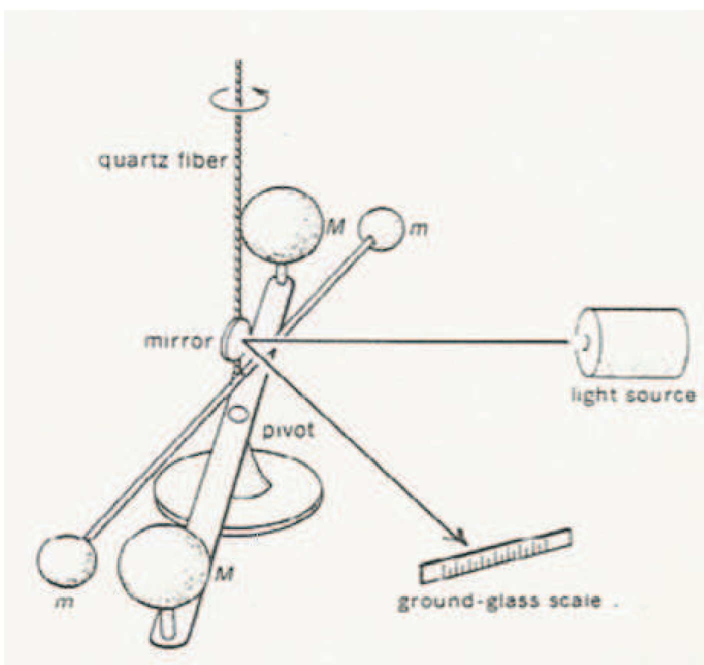
A3. In 1797-8 British scientist Henry Cavendish made the first experiment to measure the force of gravity between masses in the laboratory. His apparatus, shown in the figure, used a torsion pendulum to determine the force F_{Mm} between small masses m and nearby large masses M. The small masses are separated by a distance ℓ on the ends of a thin rod of negligible mass. This "dumbbell" is suspended in its middle by a quartz fiber with torsion constant κ. The large masses are also separated by ℓ and can be moved close to or away from the small masses by rotation of their support arm (of negligible mass) around the pivot, which is located directly middle of the dumbbell.

The force F_{Mm}, and therefore Newton's gravitational constant G, is determined by measuring the static displacement θ of the torsion pendulum when the masses are placed near to each other. When the large masses are in positions AB, the dumbbell rotates counterclockwise by θ (relative to it's relaxed position); when they are in positions $A'B'$, the dumbbell rotates clockwise by the same amount.

The locations of the masses are determined by measuring the angle θ of the dumbbell with respect to it's relaxed position and the angle ϕ of the bar supporting large masses with respect to this same axis.

In an auxiliary experiment, the period T of the torsion pendulum is measured by setting it into small oscillations (while the large masses are removed) and measuring the time for a large number of cycles.



Give your answers in terms of these measured quantities:

1. The oscillation period T of the torsion pendulum with the large masses removed.
2. The mass m and radius r of each small mass. (Identical)
3. The mass M and radius R of each large mass. (Identical)
4. The distance ℓ separating the small masses and also the large masses. (Identical)
5. The angle ϕ between the axis of the large masses and the equilibrium axis of the small masses.
6. The static displacement from equilibrium θ of the dumbbell when the large masses are at ϕ.

Hint: Only for calculating the rotational inertia of the dumbbell, you may treat the small masses as point masses a distance ℓ apart.

(a) (*2 points*) Write the equation of motion for the torsion pendulum, with the large masses removed and neglecting air resistance and internal friction in the fiber, and find the torsion constant of the quartz fiber in terms of measured quantities.

(b) (*4 points*) Write an equation for the force between the nearby large and small masses when they are close to each other (position AB) and use it to find an expression for the gravitational constant G in terms of the measured quantities. (Ignore the more distant interactions when doing this part. You may also take $\theta \ll \phi$ and make the small angle approximation for functions of ϕ.)

The remainder of the question is about corrections to and implications of your answer to part (b).

(c) (*1 point*) You can compute the densities of the spheres from their masses and radii. How small can the angle ϕ be for a given material density and given values of M and m? (You want infinitesimal distances between the sphere surfaces.)

(d) (*1 point*) Calculate the ratio of the torque on the dumbbell from the further masses to that from the near masses, using the same approximations as in part (b).

(e) (*2 points*) In his experiment, Cavendish wanted to "weigh" the earth and not to determine G. Use the facts that the radius of the earth is R_e and the local gravitational acceleration is g to find an expression for the mass of the earth in terms of the measured quantities.

$$\text{Note:} \quad \cos 2x = \cos^2 x - \sin^2 x \qquad \sin 2x = 2\sin x \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \qquad \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\begin{aligned}
\cos x + \cos y &= 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \\
\cos x - \cos y &= -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) \\
\sin x + \sin y &= 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \\
\sin x - \sin y &= 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)
\end{aligned}$$