Prelim ID Number: ______

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, 09:00–12:00, Aug 20, 2010

Instructions

(a) You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

(b) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

(c) For convenience in grading please write legibly, use only one side of each sheet of paper, and work **different problems on separate sheets of paper**. The sheets for each problem will be stapled together but separately from the other two problems.

(d) You will be assigned a **Prelim ID Number**, *different from your UF ID Number*. The **Prelim ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of **each sheet**. Do **NOT** use your name or UF ID Number anywhere on the Exam.

(e) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

(f) Each problem is worth 10 points.

(g) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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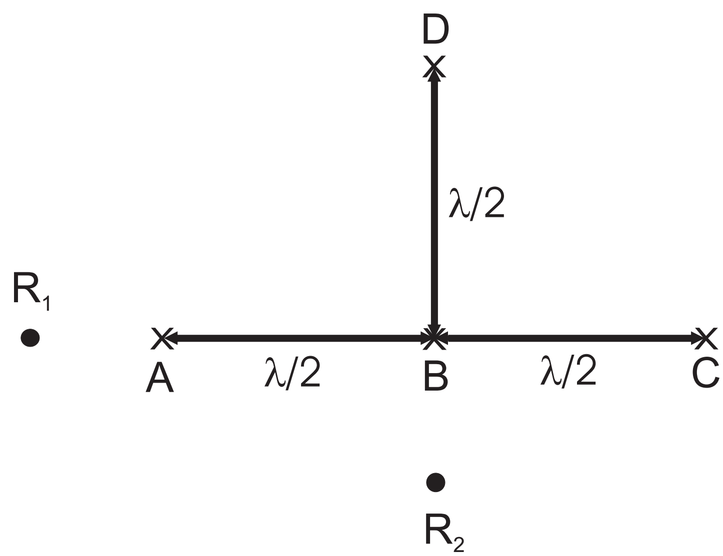
C1. Four identical sources **A**,**B**,**C** and **D**, arranged as shown in the figure, produce harmonic electromagnetic waves of the same wavelength λ, initial phase, polarization, and amplitude of the electric field E_0. Two receivers, R_1 and R_2, are at an equal distance r from B, and $r \gg \lambda$ (not drawn to scale).

(a) (*4 points*) What is the electric field at each of the receivers?

(b) (*3 points*) Which receiver will measure the greater signal intensity?

(c) (*3 points*) Source B or D can be turned off. Which receiver can determine which source, B or D, has been turned off? Hint: Assume that the waves reach the receivers with negligible energy loss.



C2. **Laser-pumped 4-level Laser System** Einstein's A and B coefficients describe the three fundamental interactions between light and atoms: Stimulated emission, stimulated absorption and spontaneous emission. He showed that the probability for stimulated emission of an excited atom to transit into the ground state is identical to the probability for stimulated absorption of a ground state atom to transit into its excited state.

Based upon his rate equations, the stimulated emission and stimulated absorption rates for the transition between states $|1\rangle$ and $|0\rangle$ are:

$$\text{stimulated Emission: } \Omega n_1 \qquad \text{Stimulated Absorption : } \Omega n_0$$

where n_1 is the population density of the higher energy state, that is the probability of each atom to be in the higher state times the density of atoms. n_0 is the population density of the lower energy state. The probability that an atom in the lower state will absorb a photon or that an atom in the upper state will emit a photon via stimulated emission in a unit amount of time,

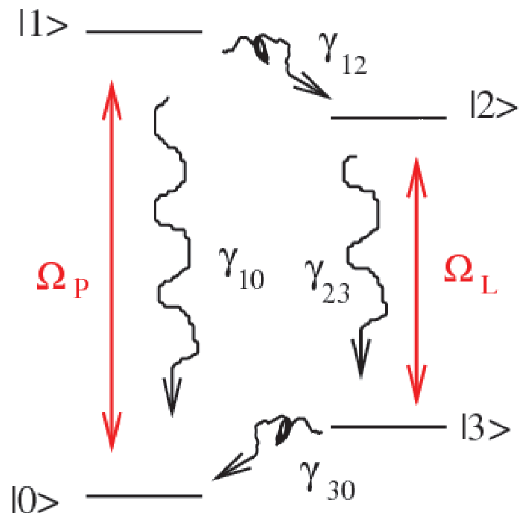
$$\Omega(\text{stimulated Emission}) = \Omega(\text{Stimulated Absorption}) = \Omega,$$

is identical for both processes. Ω is usually measured in Hz. These processes couple the laser field directly to the atomic states.

The third process, spontaneous emission, is only possible from a higher energy state into a lower state and is described by a decay rate

$$\gamma_{nm} \quad \text{with}\, n: \text{ Higher Level, } \; \text{and}\, m: \text{ Lower Level.}$$

γ_{nm} is also measured in Hz. This process emits photons in arbitrary directions out of the laser field. This is fluorescent light.



The figure shows a typical four level laser system:

Each energy-level corresponds to an atomic eigenstate with eigenenergy

$$E_1 > E_2 > E_3 > E_0.$$

An external laser pump-field stimulates transitions between atomic eigenstates $|1\rangle$ and $|0\rangle$. The probability for these transitions is proportional to Ω_P. The laser operates between levels $|2\rangle$ and $|3\rangle$. Photons in the laser field are generated when atoms undergo stimulated emission from state $|2\rangle$ to state $|3\rangle$. Photons in the laser field are absorbed when atoms transit from state $|3\rangle$ to state $|2\rangle$. The probability per unit time for the stimulated processes between states $|2\rangle$ and $|3\rangle$ is proportional to Ω_L. In addition, the laser operation also depends on the four spontaneous emission processes, described by the rates γ_{10}, γ_{12}, γ_{23} and γ_{30}. All other spontaneous emission processes are so small that we can neglect them.

(a) (*4 points*) The population densities of each state are described by n_i, $i = 0, 1, 2, 3$. Calculate the population densities n_1, n_2, n_3 as a function of the Ω's, γ's, and n_0 at steady state.

Start by setting up the rate equations for all four states. To get you started, the first one is:

$$\dot{n}_1 = \Omega_P(n_0 - n_1) - (\gamma_{12} + \gamma_{10})n_1$$

Note: Steady state means that the population densities are constant. Note also that the condition $n_0 + n_1 + n_2 + n_3 = N$, the atomic density, could be used to calculate all four densities as a function of the $\Omega's$ and $\gamma's$ but this does not lead to any additional insight into the problem.

(b) (*3 points*) Note: You don't need to have solved part (a) to answer these questions. Some physical insight should be enough.

i. If we turn off the pump field Ω_P, what would be the $1/e$-lifetime of atoms in levels $|1\rangle$, $|2\rangle$, and $|3\rangle$?

ii. A primary condition for laser operation is that the laser field E_L is amplified as it propagates through the medium. Under which of the following conditions can this be achieved? Why? Two or three sentences should be sufficient.

$$\text{Choose either} \qquad \frac{\gamma_{30}}{\gamma_{23}} \ll 1 \qquad \text{or} \qquad \frac{\gamma_{30}}{\gamma_{23}} \gg 1$$

iii. Based on part (b), we know already which of the two decay rates γ_{30} and γ_{23} should be large and which one should be small. What about γ_{12}? If you had to look for a 4-level system to create a laser, would you prefer a system in which γ_{12} is large or small? Why? One or two sentences is sufficient.

(c) (*3 points*)

i. One important characteristic of any laser system is the population inversion. This can be expressed as a product between the small signal inversion and a term describing the saturation:

$$\Delta n = n_2 - n_3 = \Delta n_0 \frac{1}{1 + \Omega_L \tau_{\text{Sat}}}$$

Calculate Δn_0 and τ_{Sat} and use the condition found in part (b.ii) to simplify the result. For this question you need to have solved part (a).

ii. Assume that $\Omega_L \tau_{Sat} = 1$. If one specific atom is in state $|2\rangle$, what is then the ratio between the probability that this atom undergoes a stimulated emission process compared to a spontaneous emission process. In other words, what is the ratio of the number of photons being emitted into the laser mode versus the number being "fluorescent light"?

C3. Each molecule of the protein myoglobin has a single site where an oxygen molecule O_2 can bind. Therefore the myoglobin can be thought of as having two states, where the number m of bound oxygen molecules is either $m = 0$ (with energy $= 0$) or $m = 1$ (with energy $= -\epsilon < 0$).

(a) (*2 points*) Imagine that this myoglobin molecule is in equilibrium with the oxygen in the atmosphere. Write down the grand partition function for the protein molecule in terms of the chemical potential μ of the atmospheric oxygen and any relevant constants.

(b) (*2 points*) Give an expression for the mean occupancy, *ie* the mean number $\langle m \rangle$ of oxygen molecules bound to one molecule of myoglobin, as a function of the atmospheric oxygen density n.

You may find it helpful to recall that atmospheric oxygen can be described as an ideal gas with density n (molecules per unit volume), so that its chemical potential may be written

$$\mu = kT \log(n/n_Q)$$

where $n_Q = (MkT/2\pi\hbar^2)^{3/2}$

(c) (*2 points*) Sketch, for myoglobin, the "oxygen binding curve", $\langle m \rangle$ vs p (the oxygen pressure).

(d) (*3 points*) The protein hemoglobin is similar to myoglobin, except that one molecule of hemoglobin can bind four oxygen molecules. We can roughly say that the hemoglobin molecule has just two states, where the number of bound oxygen molecules is either $m = 0$ (with energy $= 0$) or $m = 4$ (with energy $= -4\epsilon$).

Give an expression for the mean occupancy of the hemoglobin molecule, *ie* the mean number $\langle m \rangle$ of oxygen molecules bound to the protein, as a function of the atmospheric oxygen density n.

(e) (*1 point*) Sketch, for hemoglobin, the "oxygen binding curve," $\langle m \rangle$ vs p. Your sketch should indicate clearly how this curve differs from that of myoglobin found in part 3.