Prelim ID Number: ______

PRELIMINARY EXAMINATION

Department of Physics

University of Florida

Part B, 14:00–17:00, Aug 18, 2011

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work **different problems on separate sheets of paper**. The sheets for each problem will be stapled together but separately from the other two problems.
4. You will be assigned a **Prelim ID Number**, *different from your UF ID Number*. The **Prelim ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of **each sheet**. Do **NOT** use your name or UF ID Number anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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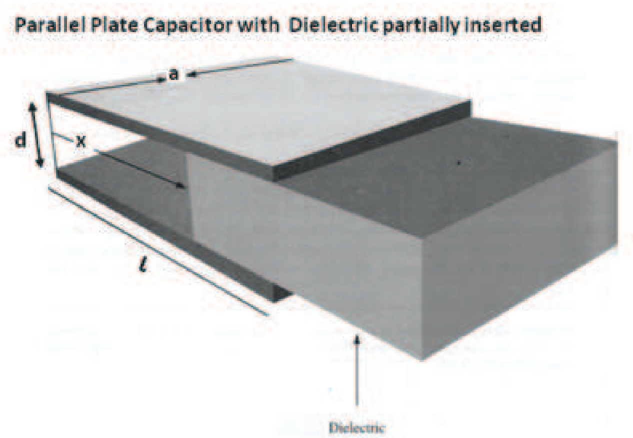
B1.



A parallel plate capacitor, with plates a distance d apart, width a and length l (see picture) has a dielectric with dielectric constant $\epsilon_r = 1 + \chi_e$, partially inserted such that x of the total length l of the dielectric remains unfilled by the dielectric. The polarization of the dielectric is assumed to be proportional to the electric field vector $\vec{E}$, $\vec{P} = \epsilon_0 \chi_e \vec{E}$, where ϵ_0 is the permittivity of free space and χ_e is the electric susceptibility.

(a) (*4 points*) Write an expression for the capacitance of the capacitor in terms of x with the dielectric partially inserted.

(b) (*2 points*) Write an expression for the energy stored in the capacitor in terms of the charge Q stored on the plates and the capacitance C you calculated from part (a). As the dielectric moves, assume that the total charge on the plates, $Q = CV$, is held constant.

(c) (*3 points*) Using your result for part (b) write an expression for the force on the dielectric as a function of the position x. Remember, Q is constant and not a function of x.

(d) (*1 points*) Is the force on the dielectric inward between the capacitor plates or is the force acting to repel the dielectric?

B2.

One form of the Clausius-Clapeyron equation of which you should be aware is $\frac{dP}{dT} = \frac{L}{T\Delta V}$, where L is the latent heat for converting a material from liquid to gas and $\Delta V = V_{\text{gas}} - V_{liquid}$, the difference in the molar volumes of the two phases. P is the pressure and T the temperature. With three straightforward assumptions, the "Vapor Pressure Equation" $P = A\exp(-L/RT)$ can be derived from this version of the Clausius-Clapeyron equation. Here A is a constant.

(a) (*0 points*) Just for fun, try to write down the three assumptions you will need before you look at them given below.

(b) (*10 points*) Derive the vapor pressure equation as stated above using the following assumptions:

i. $L =$ constant.

ii. The molar volume of the liquid phase is small, so that $\Delta V = V_{\text{gas}}$.

iii. $V_{\text{gas}} = RT/P$.

B3.

Consider a nonrelativistic quantum particle of mass m scattering off an isotropic finite range potential $V(r)$. The scattering amplitude $f(\theta)$ is defined by the large r behavior of the wave function in the potential:

$$\psi \sim C\left(e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right), \qquad \text{as } r \to \infty, \tag{1}$$

where ψ satisfies the time independent Schrödinger equation with energy eigenvalue $E = \hbar^2 k^2/2m$. The plane wave e^{ikz} has the Legendre polynomial expansion

$$e^{ikz} = e^{ikr\cos\theta} = \sum_{l=0}^{\infty}(2l+1)i^l j_l(kr)P_l(\cos\theta) \tag{2}$$

Recall that the Legendre polynomials are normalized so that $P_l(1) = 1$ and the spherical Bessel functions have the large r behavior

$$j_l(kr) \sim \frac{\sin(kr - l\pi/2)}{kr}, \qquad \text{as } r \to \infty, \tag{3}$$

(a) (*2 points*) Explain why the solution ψ may be expanded in Legendre polynomials as follows:

$$\psi = \sum_{l=0}^{\infty}(2l+1)i^l R_l(r)P_l(\cos\theta) \tag{4}$$

and write down the radial differential equation satisfied by $R_l(r)$.

(b) (*3 points*) The scattering phase shift for the potential V is defined by the large r behavior of the radial wave function

$$R_l(r) \sim A\frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}, \qquad \text{as } r \to \infty, \tag{5}$$

Prove that the scattering amplitude is given in terms of the phase shifts as

$$f(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(e^{2i\delta_l} - 1)(2l+1)P_l(\cos\theta) \tag{6}$$

(c) (*2 points*) Specializing to scattering from a hard sphere of radius a (i.e. $V = \infty$ for $r < a$, and $V = 0$ for $r > a$) calculate exactly the s-wave phase shift $\delta_0(k)$.

(d) (*3 points*) For finite range potentials, $\delta_l/\delta_0 = O(k^{2l})$ as $k \to 0$, which you may assume without proof. Then calculate the low energy ($ka \ll 1$) differential and total cross sections for scattering off a hard sphere.