Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, August 2012, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

6. Each problem is worth 10 points.

7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. A harmonic oscillator of mass m and angular frequency ω is in a state $\Psi(x,t)$ such that a measurement of the energy would yield either $(1/2)\hbar\omega$ or $(3/2)\hbar\omega$, with equal probability.

(a) [2 points] Write down the most general form for the state $\Psi(x,t)$ in terms of the harmonic oscillator wave functions and eigenvalues, consistent with the given condition.

(b) [6 points] What is the largest possible value for $\langle p \rangle$ in the above state? Here p is the momentum operator and $\langle \ \rangle$ represents the expectation value. [Note: The raising and lowering operators $a^\dagger$ and a, with properties $a^\dagger \psi_n = \sqrt{n+1}\psi_{n+1}$ and $a\psi_n = \sqrt{n}\psi_{n-1}$, are defined as $a^\dagger \equiv \frac{1}{2\hbar m\omega}(-ip + m\omega x)$ and $a \equiv \frac{1}{2\hbar m\omega}(ip + m\omega x)$ where p and x are the momentum and position operators, respectively.]

(c) [2 points] If it assumes the maximum value of $\langle p \rangle$ at time $t = 0$, what is $\Psi(x,t)$?

A2. Consider a standard one-dimensional square well,

$$V(x) = \begin{cases} 0, |x| & > a \\ -V_0, |x| & \leq a \end{cases}$$

(a) [2 points] Write down solutions of Schrödinger equation for a particle of mass m in all regions with energy $E > 0$.

(b) [4 points] Particles of energy $E > 0$ are incident on it from the left. Calculate the transmission coefficient T.

(c) [2 points] How does T behave for very large energies? What is its low-energy limit?

(d) [2 points] Are there any specific values of positive energy for which there is absolutely no reflection and the well is transparent? Verify explicitly that for these particular values the amplitude of the reflected wave vanishes.

(Note: (c) and part of (d) can be completed without completion of (b) from sound physical reasoning.)

A3. Consider the quantum mechanical bound states of a one dimensional particle of mass m which moves in the linear potential of the Earth's gravitational field,

$$V(x) = \begin{cases} mgx & \text{for } x > 0 \\ \infty & \text{for } x < 0 \end{cases}$$

This problem concerns using the WKB method to approximate the energy eigenstates. It happens that there is an exact solution to this problem and you will receive full credit if you find it, but we will not give you the special functions.

(a) [2 points] What is the form the ground state energy from dimensional analysis?

(b) [3 points] What are the classical turning points?

(c) [2 points] What is the WKB quantization condition for this problem?(Just write the integral expression.)

(d) [2 points] What are the WKB energy eigenvalues?