Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, August 2012, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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C1. A solid contains N chlorine ions located on hcp lattice. The Cl nuclei have nuclear spin $I = 3/2$ and in the solid the nuclei have an interaction Hamiltonian

$$H = \hbar\omega(3I_z^2 - I^2)$$

due to the interaction of the nuclei with electric field of the lattice.

(a) [2 points] Draw the energy level scheme for the nuclei showing the energy values and the degeneracies.

(b) [2 points] Find a simple closed expression for the partition function.

(c) [2 points] Calculate the internal energy U.

(d) [2 points] Calculate the heat capacity C as a function of temperature. Draw a sketch of C(T) as a function of T.

(e) [2 points] Explain in words why the heat capacity vanishes at both very low and very high temperatures.

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C2. In this problem, we will look at some aspects of *Black Body Radiation*.

(a) [1 point] Black body radiation can be thought of as an equilibrium gas of photons. What type of statistics do photons obey?

(b) [2 points] How does the density of states per unit volume $g(\omega)$ depend upon $\omega = E/\hbar$ for a three dimensional photon gas?

(c) [3 points] Using (a) and (b) above, show that the energy density distribution function $u(\omega)$ (*i.e.*, the energy per unit volume in the interval between ω and $\omega + d\omega$) is given by

$$u(\omega) = \frac{A(\hbar\omega)^3}{\exp(\hbar\omega/k_B T) - 1}$$

Where A is a constant (which you *do not* need to determine). Hence show that the total energy radiated per unit volume $U = \int d\omega \; u(\omega)$ is proportional to T^4. Note that you do not have to calculate the proportionality constant.

(d) [2 points] The *power radiated per unit area* from the surface of a black body radiator is given by $R = \frac{c}{4}U = \sigma T^4$ where c is the speed of light and $\sigma = 5.7 \times 10^{-8}$ W/m^{2}K^4 is Stefan's constant.

From this result, calculate the average power per unit area at the top of the earth's atmosphere. You might want to use the facts that the surface temperature of the sun is 5800 K, the radius of the sun is $r_{sun} = 7.0 \times 10^8$m, the radius of the earth is $r_{earth} = 6.4 \times 10^6$m and the distance from the earth to the sun is 1.5×10^{11}m. This number is very important in the area of photovoltaics and solar energy.

(e) [2 points] From the results in part (d), calculate the black body temperature of the earth. Assume that: 1) none of the sun's energy is reflected by the atmosphere and 2) the earth is a perfect black body that radiates equally in all directions. (Neither of these assumptions is accurate).

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C3. Consider a sphere of uniform mass density ρ and radius R. Its gravitational field (i.e. the gravitational force on a test particle per unit test particle mass) has the form $\vec{g}(\vec{r}) = \hat{r}g(r)$ where $\vec{r}$ is position relative to the center of the sphere and $\hat{r} = \vec{r}/r$ is the unit vector in the direction of $\vec{r}$.

(a) [points] What is the function $g(r)$ for $r > R$?

(b) [points] What is the function $g(r)$ for $r < R$?

Assume that the sphere is made up of infinitely many 'dust' particles, *i.e.*, point particles that have no interactions other than gravity. At initial time $t = 0$, the sphere is in a state of uniform expansion, *i.e.*, the particle at position $\vec{r}$ has velocity $\vec{v} = H\vec{r}$. H is called the expansion rate.

(c) [points] Obtain the equation of motion for the radius $R(t)$ of the sphere as a function of time.

(d) [points] There is a critical value H_c of the expansion rate such that for $H < H_c$ the sphere will start to contract after some time, whereas for $H > H_c$ the sphere expands for ever. Obtain H_c as a function of ρ.

(e) [points] Obtain the radius $R(t)$ as a function of time for the special case $H = H_c$.