Student ID Number: ________

PRELIMINARY EXAMINATION
Department of Physics
University of Florida
Part A, August, 2013, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. A bead of mass m is constrained to move on a frictionless circular wire hoop of radius R, which is forced to rotate with angular velocity ω about its fixed vertical diameter. Gravity is a uniform downward force on the bead. With the origin of coordinates at the fixed center of the hoop, use the angle θ, between the vertical and the bead's position vector, as the generalized coordinate of the bead.

(a) **[2 points]** Write down the Lagrangian describing the dynamics of the bead.

(b) **[2 points]** Identify the conserved quantities, if any, for this system.

(c) **[2 points]** For $\omega > \sqrt{g/R}$, find the constant values $\theta_0 \neq 0, \pi$ for which $\theta(t) = \theta_0$ is a static solution of the equations of motion.

(d) **[2 points]** Show that the static equilibrium solution found in part (c) is stable.

(e) **[2 points]** Find the angular frequency ω_0 of small oscillations about the static solution of part (c).

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A2. A charged particle with mass m and charge q is attached to a massless pendulum of length ℓ. The pendulum performs small oscillations whose amplitude is decreasing because of radiation emitted by the charge. Assume that the radiation losses, ΔE, during one oscillation are small compared to the mechanical energy E of the pendulum.

(a) [**1 point**] What is the angular frequency ω of pendulum's oscialltions, if the gravity of Earth is g?

(b) [**3 points**] If pendulum's oscillations expressed for the angle θ between the pendulum and the vertical are $\theta(t) = \theta_0 \, sin(\omega t)$, what are power $P(t)$ of the energy lost to radiation?

(c) [**2 points**] What is the total energy lost for one full forth and back oscilation with an amplitude θ_0?

(d) [**4 points**] Find how the amplitude of oscillations θ_0 changes with time due to energy losses on radiation.

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A3. **Precessing spin**. The Hamiltonian for a spin 1/2 particle in a uniform magnetic field $B = B_0\hat{z}$ may be written $\mathcal{H} = -\mu\vec{\sigma}\cdot\vec{B}$ where $B = B_0\hat{z}$ and $\vec{\sigma}$ is the Pauli spin vector. At time $t = 0$ the spin is along the $\hat{x}$ direction (an eigenfunction of σ_x with eigenvalue $+1$).

(a) [**3 points**] Solve the time dependent Schrödinger equation to find the spin wave function at time t.

(b) [**3 points**] Find the expectation value of σ_x at time t. What is the spin precession frequency?

Hint: The Pauli matrices are:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(c) [**2 points**] The field is now changed to

$$\begin{aligned} B_x &= B_1 \cos\omega t \\ B_y &= B_1 \sin\omega t \\ B_z &= \mathit{const.} \gg B_1 \end{aligned}$$

What value of ω gives resonant transitions (takes an up spin and converts it to a down spin with maximum probability)?

(d) [**2 points**] At $t = 0$ all the particles are polarized in the state $|\uparrow\rangle$. What is the probability for a particle at time t to have spin in the $-z$ direction?