Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, August, 2013, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

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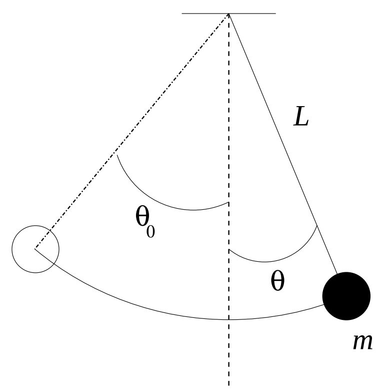
A1. Consider a plane pendulum consisting of a particle of mass m hanging from a rigid support by a massless rigid rod of length L as shown in the figure. Force of gravity acts downward. Ignore all frictional forces.



(a) **[3 points]** Write down the equation of motion for the amplitude θ. What is the period of oscillation if θ is restricted to small values such that $\sin\theta \approx \theta$?

In the following, we wish to consider a more general case where the amplitude is NOT small. Suppose initially at time $t = 0$ the mass is at rest at $\theta(t = 0) = \theta_0$ where $-\pi < \theta_0 < \pi$.

(b) **[2 points]** Write down the kinetic and the potential energies of the system at any later time in terms of θ and $\dot{\theta} \equiv d\theta/dt$.

(c) **[3 points]** Using energy considerations, obtain $\dot{\theta}$ in terms of θ and θ_0 and obtain an expression for the period as an integral over θ.

(d) **[2 points]** Expand the integrand for small θ_0 up to order θ_0^2 to obtain the leading correction to the period for small amplitude oscillation obtained in (a).

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B1. A region of space has electric and magnetic fields with cylindrical symmetry, and thus we use cylindrical coordinates (ρ, ϕ, z) where $\rho = \sqrt{x^2 + y^2}$ and $\tan\phi = y/x$. In this region, the electric potential is $V = -(a\rho)/\varepsilon_0$, and the magnetic vector potential is $\vec{A} = -\mu_0 b\rho^3 \hat{z}$, where a and b are constants and ε_0 and μ_0 are the usual permittivity and permeability constants.

(a) [**2 points**] What is the electric field?

(b) [**2 points**] What is the electric charge density?

(c) [**2 points**] What is the magnetic field?

(d) [**2 points**] What is the current density, including direction?

(e) [**2 points**] Give a modified magnetic vector potential such that the magnetic field would have the same direction as in part (c) but a constant magnitude.

Vector Calculus in cylindrical coordinates:

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \tag{1}$$

$$\nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \tag{2}$$

$$\nabla \times \vec{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\phi} + \frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)\hat{z} \tag{3}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \tag{4}$$

$$\nabla^2 \vec{A} = \left(\nabla^2 A_\rho - \frac{A_\rho}{\rho^2} - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial \phi}\right)\hat{\rho} + \left(\nabla^2 A_\phi - \frac{A_\phi}{\rho^2} + \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial \phi}\right)\hat{\phi} + (\nabla^2 A_z)\hat{z} \tag{5}$$

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D3. Consider a Carnot engine operating between two heat reservoirs at $T = T_1$ and $T = T_0$ as shown in Figure 1. This engine absorbs heat Q from the hot reservoir at T_1 and delivers heat Q' to the cold reservoir T_0. The efficiency η of this engine is $1 - T_0/T_1$.

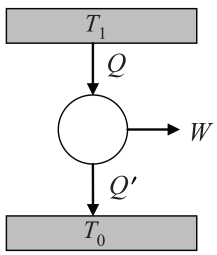

Figure 1

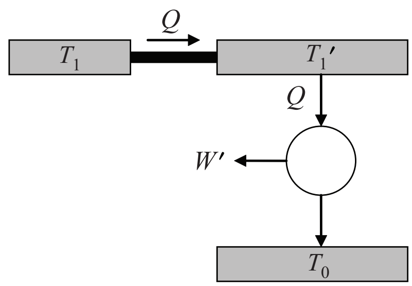

Figure 2

(a) [**1 point**] Express the work W produced by this reversible engine in one cycle in terms of Q, T_1 and T_0.

(b) [**1 point**] What is the change in the entropy ΔS_{engine} after one cycle. State your reasoning.

(c) [**1 point**] Calculate the change in the entropy $\Delta S_{\text{universe}}$ where the universe comprises the high temperature reservoir and the low temperature reservoir. What is the change in total entropy, $\Delta S_{\text{total}} = \Delta S_{\text{engine}} + \Delta S_{\text{universe}}$?

(d) [**2 points**] Let us put the same amount of heat to the same Carnot engine from a second reservoir at $T_1'(< T_1)$ s shown in Figure 2 by allowing the heat Q to be conducted from the reservoir at T_1 to the reservoir at T_1' through a conducting bar. Obviously there is an irreversible process inserted here. What is the work produced from the engine W' in a cycle? From your answer, calculate $\Delta W = W - W'$.

(e) [**2 points**] Calculate the change in the total entropy $\Delta S_{\text{total}} = \Delta S_{\text{engine}} + \Delta S_{\text{universe}}$ for this second case.

(f) [**3 points**] Find the direct relationship between the difference in work produced by these engines ($\Delta W = W - W'$) and the difference in the total entropy changes between the scenarios depicted in Figure 1 and Figure 2. This result is often called the degradation of energy. Can you argue anything about the fate of the universe based on this simple result in which the difference in work available is related to the difference in total entropy change between Figures 1 and 2?