

Student ID Number: \_\_\_\_\_

**PRELIMINARY EXAMINATION**

DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

Part D, August, 2013, 14:00–17:00

**Instructions**

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

**University of Florida Honor Code:** We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *“On my honor, I have neither given nor received unauthorized aid in doing this assignment.”*

**DO NOT OPEN EXAM UNTIL INSTRUCTED**

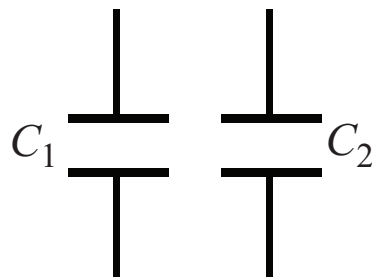
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- D1. Consider the two isolated capacitors shown in the figure. Capacitor #1 has capacitance  $C_1 = 1\mu\text{F}$  and capacitor #2 has capacitance  $C_2 = 2\mu\text{F}$ . Suppose that you have a total charge  $Q_{\text{tot}} = 30\mu\text{C}$  that must be stored on the two capacitors, with  $Q_1$  placed on  $C_1$  and  $Q_2$  placed on  $C_2$  such that  $Q_1 + Q_2 = Q_{\text{tot}}$ .



- (a) [**3 points**] How much electric charge,  $Q_1$  and  $Q_2$  (in  $\mu\text{C}$ ), would you place on each capacitor to maximize the total amount of stored energy,  $U_{\text{tot}} = U_1 + U_2$ , and what is  $U_{\text{tot}}(\text{max})$  (in  $\mu\text{J}$ )?
- (b) [**4 points**] How much electric charge,  $Q_1$  and  $Q_2$  (in  $\mu\text{C}$ ), would you place on each capacitor to minimize the total amount of stored energy,  $U_{\text{tot}} = U_1 + U_2$ , and what is  $U_{\text{tot}}(\text{min})$  (in  $\mu\text{J}$ )?
- (c) [**3 points**] What are the potential drops,  $\Delta V_1$  and  $\Delta V_2$  (**in volts**), across each of the two capacitors in (b)?

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D2. Consider a particle of mass  $\mu$  moving in three dimensions. Its Hamiltonian is  $H = \frac{1}{2\mu} \vec{P} \cdot \vec{P} + V(\vec{X})$ .

(a) [ **points** ] Show that

$$\frac{d}{dt} \langle \Psi'(t) | \vec{X} \cdot \vec{P} | \Psi(t) \rangle = \langle \Psi'(t) | \left[ \frac{1}{\mu} \vec{P} \cdot \vec{P} - \vec{X} \cdot \vec{\nabla} V(\vec{X}) \right] | \Psi(t) \rangle \quad (1)$$

for any pair of states  $|\Psi\rangle$  and  $|\Psi'\rangle$ .

(b) [ **points** ] Show that

$$\langle E | \frac{1}{\mu} \vec{P} \cdot \vec{P} | E \rangle = \langle E | \vec{X} \cdot \vec{\nabla} V(\vec{X}) | E \rangle \quad (2)$$

in any eigenstate  $|E\rangle$  of the Hamiltonian.

(c) [ **points** ] Assume that  $V(\vec{X}) = c|\vec{X}|^\gamma$  where  $c$  and  $\gamma$  are constants. Prove the ‘quantum virial theorem’, i.e. that in any energy eigenstate

$$\langle T \rangle = \frac{\gamma}{2} \langle V \rangle \quad (3)$$

where  $\langle T \rangle$  and  $\langle V \rangle$  are the expected values of the kinetic and potential energies.

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- D3. A laser beam with wavelength 632 nm is incident on a gas of  $\text{H}_2$  molecules. A small fraction of the photons gain or lose energy when they are scattered inelastically by excitations of the  $\text{H}_2$  molecules (Raman effect). Consider scattering by the fundamental ( $n = 0$  to  $n = 1$ ) vibrational excitation, which has an energy of  $8.3 \times 10^{-20}\text{J}$ .
- (a) **[2 points]** Calculate the wavelength,  $\bullet_+$ , of the scattered photons (in nm) when they gain the energy corresponding to the transition of the  $\text{H}_2$  molecule from the first excited state ( $n = 1$ ) to the ground state ( $n = 0$ ).
- (b) **[2 points]** Now calculate the wavelength,  $\bullet_-$ , of the scattered photons (in nm) if they instead lose this amount of energy to an  $\text{H}_2$  molecule by transitioning the molecule from the ground state into the first excited vibrational state.
- (c) **[3 points]** At room temperature, do you expect the number of photons with wavelength  $\bullet_+$  to be much more, much less, or about the same as the number of photons with wavelength  $\bullet_-$ ? Why?
- (d) **[3 points]** By approximating the system as quantum mechanical harmonic oscillator, use the energy of the fundamental ( $n = 0$  to  $n = 1$ ) vibrational excitation ( $8.3 \times 10^{-20}\text{J}$ ) to estimate the spring constant of the  $\text{H}_2$  molecule (in N/m).