Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, August 17, 2021, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. **The sheets for each problem will be stapled together but separately from the other two problems**.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. A particle of mass m moves in the one dimensional potential

$$
\begin{aligned}
V(x) \qquad &= 0 \text{ for } x > a \\
&= -|V_o| \text{ for } 0 < x < a \\
&= \infty \text{ for } x < 0.
\end{aligned}
$$

The particle is in a state of energy E in the range $-|V_o| < E < 0$.

(a) [**2 points**] What is the general form of the solution for $0 < x < a$?

(b) [**2 points**] What is the general form of the solution for $x > a$?

(c) [**3 points**] What are the boundary conditions at $x = 0$, $x = a$, $x \to \infty$?

(d) [**3 points**] Apply the boundary conditions to determine the range of $|V_o|a^2$ for which there is a solution. (A graphical method is the most common way to do this.)

A2. A spinless particle of mass μ moves in the (x, y) plane under the influence of an axially symmetric potential $V(r)$. (r, ϕ) are polar coordinates in the plane.

(a) **[2 points]** Show that the angular momentum of the particle acts as the operator

$$L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi} \tag{1}$$

on the particle's position-space wavefunction $\psi(r, \phi)$.

(b) **[2 points]** Obtain the eigenfunctions and eigenvalues of L_z.

(c) **[2 points]** At time $t = 0$, the particle is in the state of wavefunction

$$\psi(r, \phi; 0) = Ae^{-\frac{r^2}{2\Delta^2}} \cos^2\phi \tag{2}$$

If l_z is measured in this state, what are the possible outcomes and what are the probabilities of those outcomes?

(d) **[2 points]** Assume the wavefunction is $\psi(r, \phi; 0)$ at time $t = 0$ but the angular momentum is measured at a later time t. What are the possible outcomes and their probabilities in this case?

(e) **[2 points]** Assume the potential $V(r, t)$ is axially symmetric and time-dependent. Do your answers in part (d) still hold?

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A3. **Harmonic Oscillator**. A particle of mass m moves in a one-dimensional harmonic oscillator potential with frequency ω. At time $t = 0$, the particle is in the state:

$$|\Psi\,(0)\rangle = A\,(\;12\;\;|n=1\rangle\;+\;5\;\;|n=2\rangle\;) \tag{3}$$

where $|n\rangle$ are the number states of the Harmonic Oscillator.

(a) **[1 point]** What is A?

(b) **[2 points]** Construct $|\Psi(t)\rangle$ and $|\Psi(t)|^2 = \langle\Psi(t)|\Psi(t)\rangle$.

(c) **[2 points]** If one measures the energy of this particle, what values would you get and with what probabilities?

(d) **[2 points]** Calculate, the expectation value of the position (as a function of time) $\langle x(t)\rangle$.

(e) **[3 points]** Calculate the expectation value of the kinetic energy operator $\left\langle \widehat{KE}(t) \right\rangle$.

Useful Formulae:

$$\begin{aligned} a_+ &= \sqrt{\frac{m\omega}{2\hbar}}\;X - i\frac{1}{\sqrt{2\hbar m\omega}}\;P \qquad a_- = \sqrt{\frac{m\omega}{2\hbar}}\;X + i\frac{1}{\sqrt{2\hbar m\omega}}\;P \\ a_+\,|n\rangle &= \sqrt{n+1}\,|n+1\rangle \qquad\qquad a_-\,|n\rangle = \sqrt{n}\,|n-1\rangle \end{aligned} \tag{4}$$