Student ID Number: ________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, August 18, 2021, 9:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. **The sheets for each problem will be stapled together but separately from the other two problems**.

   (c) Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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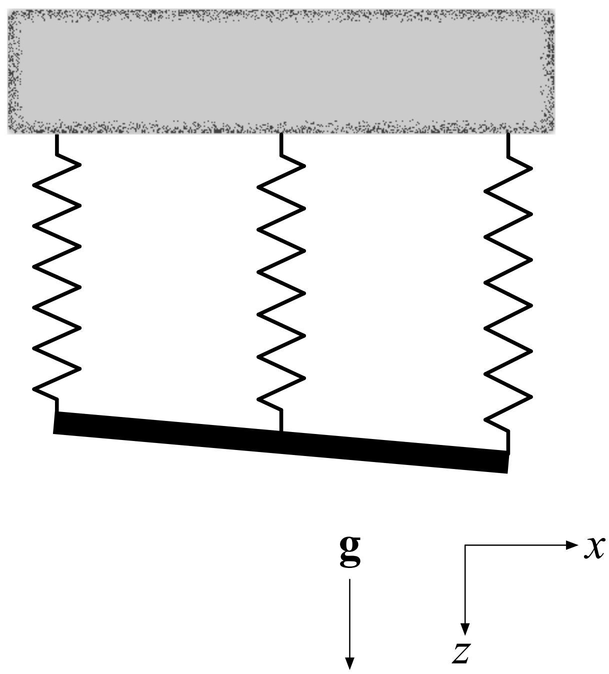
C1. As shown in the figure, a long uniform bar of length d and mass M hangs from three springs, each with unstretched length L and spring constant k. The springs are attached at the left end, the center and the right end of the bar.

Consider only displacements in the z direction and small rotations in the $x - z$ plane (i.e., ignore any "swinging" motions). For a long bar, the moment of inertia about the center is $\frac{1}{12}Md^2$.



(a) **[2 points]** Find z_{eq}, the equilibrium position of the bar. z_{eq} is measured relative to the position of the unstretched springs.

(b) **[3 points]** Consider center-of-mass oscillations in the z direction while the bar remains horizontal. Write the equation of motion of the center of mass, z_{CM}, where z_{CM} is measured relative to the equilibrium position. What is the oscillation angular frequency in terms of M, d, k, L, g?

(c) **[3 points]** Consider small oscillations where the center-of-mass is fixed and the bar rotates in the $x - z$ plane. Write the equation of motion in terms of θ, the (small) angle from the horizontal. What is the oscillation angular frequency in terms of M, d, k, L, g?

(d) **[2 points]** Let z_{max} be the amplitude of the motion in part (b) and θ_{max} be the amplitude of the motion in part (c). What is θ_{max} in terms of z_{max} if the total energies of motion are equal in the two cases?

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DEPARTMENT OF PHYSICS
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C2. Use the following information to examine how the gravitational acceleration $g(r)$ due to Earth's gravity varies with radius. Parts **(a) - (c)** consider $g(r)$ *above* the Earth's surface ($r \geq R_E$), and parts **(d) & (e)** consider $g(r)$ *within* the Earth ($r < R_E$).

Total mass of Earth: $M_E = 5.97 \times 10^{24}$ kg
Total radius of Earth: $R_E = 6.37 \times 10^6$ m
Mass of Earth's core: $M_c = 1.93 \times 10^{24}$ kg
Radius of Earth's core: $R_c = 3.49 \times 10^6$ m

(a) [**2 points**] Use the above information to calculate $g(R_E)$. Then find the distance $r > R_E$ from the Earth's center of mass where an orbiting satellite would feel a gravitational acceleration equal to half of this value ($g(r) = g(R_E)/2$).

(b) [**1 point**] Find the speed of a satellite orbiting the Earth at the radius found in part **(a)**, assuming a circular orbit.

(c) [**2 points**] Now consider a rocket launched from the surface of the Earth. Neglecting air resistance, what minimum speed is required for the rocket to escape the Earth's gravitational potential?

(d) [**2 points**] The interior structure of the Earth can be assumed to consist of two concentric layers: a uniform-density spherical core of mass M_c and radius R_c, surrounded by the "mantle," a spherical shell with a different uniform density. (You may neglect the thin outer "crust" layer of the Earth.) Using the above information, find the gravitational acceleration $g(R_c)$ at the Earth's core-mantle interface.

(e) [**3 points**] Write an expression for how the gravitational acceleration $g(r)$ varies with radius within the mantle ($R_c \leq r \leq R_E$). Show that $g(r)$ has a local minimum within the mantle. Calculate the radius at which this minimum occurs and the corresponding minimum value of $g(r)$.

C3. An atom has two non-degenerate energy levels: a ground state of zero energy and an excited state of energy Δ.

(a) [**2 points**] Show that the partition function Z_{atom} is given by

$$Z_{\text{atom}} = 1 + e^{-\beta\Delta}$$

where $\beta = \frac{1}{k_B T}$.

(b) [**2 points**] Using the result from part (a), show that the internal energy of the atom is given by

$$U_{\text{atom}} = \frac{\Delta e^{-\beta\Delta}}{1 + e^{-\beta\Delta}} \ .$$

(c) [**2 points**] Hence, show that the heat capacity of the atom is

$$C_{\text{atom}} = \frac{\Delta^2 e^{-\beta\Delta}}{k_B T^2 \left(1 + e^{-\beta\Delta}\right)^2} \ .$$

(d) [**2 points**] Find the partition function for a monatomic gas of N such (indistinguishable) particles contained in a volume V. Given: the N-particle partition function for an ideal gas is $Z_N = \frac{1}{N!}\left(\frac{V}{\lambda_{\text{th}}^3}\right)^N$, where $\lambda_{\text{th}} = h\sqrt{\frac{\beta}{2\pi m}}$.

(e) [**2 points**] Show that the internal energy of a monatomic gas of N such (indistinguishable) particles is

$$U = \frac{N\Delta e^{-\beta\Delta}}{1 + e^{-\beta\Delta}} + \frac{3}{2} N k_B T \ .$$