Instructions

1. You may use a non-programmable calculator (i.e. one that cannot store formulas). No other tables or aids are allowed or required.

   (a) All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

   (b) For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

   (c) Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.

   (d) All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

   (e) Each problem is worth 10 points.

   (f) Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: “On my honor, I have neither given nor received unauthorized aid in doing this assignment.”

DO NOT OPEN EXAM UNTIL INSTRUCTED
B1. A rectangular loop of wire of length $L$ and width $W$ is embedded in a constant magnetic field $\mathbf{B} = B\mathbf{\hat{z}}$, as shown in the figure below at time $t = 0$. The wire itself may be considered perfectly conducting, but there is a resistor of resistance $R$ in the loop. The crank is used to rotate the loop about the dashed line ($y$-axis) in the direction shown with a constant angular frequency $\omega$, such that the angle in radians between the normal to the plane of the loop and the $\mathbf{B}$ field is $\omega t$.

(a) [2 points] Calculate the magnetic flux through the loop as a function of time.

(b) [2 points] Calculate the induced emf in the loop as a function of time.

(c) [2 points] Calculate the current flowing in the loop as a function of time. Give the direction of the current at time $t = \frac{\pi}{2\omega}$.

(d) [2 points] Determine the average power that must be supplied at the crank to maintain the rotation of the loop.

(e) [2 points] What is the force on the leftmost section of the wire (as it appears in the figure) at time $t = \frac{\pi}{2\omega}$ (magnitude and direction)?
B2. A sphere of radius $R$ has a spherical cut-out; the center of the cut-out is at distance $\vec{a}$ away from the center of the sphere. The sphere has a uniform 3D charge density $\rho$. The cut-out has a radius $s$, is devoid of charged material and its size is such that $a + s < R$. Following the coming steps, your task will be to determine the electric potential $V$ at an arbitrary point $\vec{r}'$ inside the cutout and along $\vec{a}$, as shown in the figure.

(a) [5 points] Derive an expression for the electric potential inside a uniformly charged sphere of radius $\tilde{R}$ using the electric field of a uniformly charged sphere as your starting point.

(b) [5 points] Breaking down the problem into a sum of highly symmetric parts, find the electric potential at point $\vec{r}'$ for the sphere described above.
B3. Suppose the electric field is

\[ \vec{E}(r, \theta, \phi, t) = A \sin \theta \left[ \cos(kr - \omega t) \frac{\cos(kr)}{r} - \sin(kr - \omega t) \frac{\sin(kr)}{kr^2} \right] \hat{\phi}, \quad \text{with} \quad \frac{\omega}{k} = c. \]

(a) [3 points] Assuming that the scalar potential is zero, what is the vector potential \( \vec{A}(r, \theta, \phi, t) \)?

(b) [3 points] What is the associated magnetic field \( \vec{B}(r, \theta, \phi, t) \)?

(c) [2 points] Calculate the Poynting vector \( \vec{S}(r, \theta, \phi, t) \).

(d) [2 points] What is the time-averaged power radiated to infinity?

Note: In spherical coordinates:

Gradient operator \( \nabla = \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{\partial}{r \sin \theta \partial \theta} + \hat{\phi} \frac{\partial}{\partial \phi} \)

Unit vectors:

\[ \hat{r} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} \]

\[ \hat{\theta} = \cos \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y} - \sin \theta \hat{z} \]

\[ \hat{\phi} = - \sin(\phi) \hat{x} + \cos(\phi) \hat{y} \]