Student ID Number: __________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part A, 4 January 2007, 09:00–12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

   University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. A charge +e, spin-$\frac{1}{2}$ particle with gyromagnetic ratio g is initially in an eigenstate $|\psi\rangle$ of $\hat{S}_z$ corresponding to eigenvalue $+\hbar/2$.

(a) (*2 points*) Evaluate the expectation value μ of the magnetic moment operator $\hat{\mu} = (ge/2m)\hat{\mathbf{S}}$ in this state. In which direction does it point?

(b) (*2 points*) What is the probability of obtaining a value of $\hbar/2$ if a measurement of $\hat{S}_x$ is made on this state?

(c) (*3 points*) At $t = 0$ a homogeneous magnetic field B_0 is applied in the y-direction. Show that the time evolution operator for this system may be expressed in the form

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle, \quad \hat{U}(t) = \cos\theta + i\sigma_y \sin\theta$$

where σ_y is a Pauli matrix, and find the form of $\theta(t)$.

(d) (*3 points*) Determine the precession period T, defined as the time for the time-evolved state to again be an eigenstate of $\hat{S}_z$, and find the form of the state $|\psi(t)\rangle$ after a time $t = T/4$. In which direction does the magnetic moment μ point now?

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DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

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A2. An infinite wire (of infinitesimal thickness) carries a constant current I up the z axis. A magnetic dipole $\mathbf{m} = m\hat{\mathbf{y}}$ is located at a distance d from the origin on the positive x axis.

(a) (*2 points*) Taking the fundamental dimensions to be mass, length, time and charge, what are the dimensions of the current I, the magnetic dipole moment m, the magnetic permeability of free space μ_0 and magnetic field $\mathbf{B}$? Answer this question in whatever electrodynamics unit scheme — MKS, Gaussian, Lorentz-Heaviside, etc — you use for the remainder of the problem, and state the system you are using.

(b) (*2 points*) What is the magnetic field $\mathbf{B}_{\text{wire}}$ due to the wire at every point in space?

(c) (*2 points*) What is the magnetic field $\mathbf{B}_{\text{dipole}}$ due to the dipole at every point in space?

(d) (*2 points*) What is the force exerted on the dipole by the wire?

(e) (*2 points*) What is the torque (about its position) exerted on the dipole by the wire?

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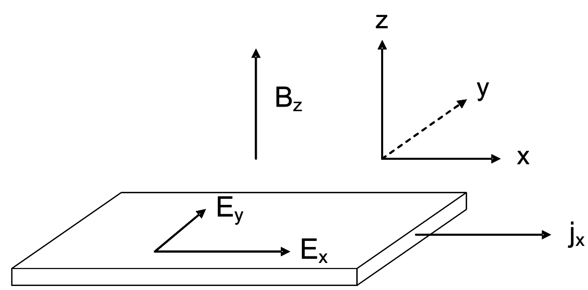
A3. Electron transport in metals can be modeled using the momentum relaxation approximation $d\mathbf{p}(t)/dt = -\mathbf{p}(t)/\tau + \mathbf{f}(t)$, which is essentially Newton's law with an external force $\mathbf{f}(t)$ and a frictional damping term inversely proportional to a scattering time τ. The momentum for an electron with mass m is equal to $m\mathbf{v}$ and the current density $\mathbf{j}(t) = -ne\mathbf{v}$, where n is the electron density and the negative sign is due to the fact that electrons carry negative charge (e is a positive quantity).



(a) (*1 point*) Assume that a time dependent electric field $\mathbf{E}(t)$ is applied, find an equation that relates $\mathbf{E}(t)$ to $\mathbf{j}(t)$ in the limit $\tau \longrightarrow \infty$ where damping can be ignored.

(b) (*1 point*) Identify the type of circuit element your answer in (a) represents (resistor, capacitor, inductor, or diode).

(c) (*2 points*) Assume a harmonic time dependence $e^{-i\omega t}$ and find the complex "conductivity" coefficient that relates $\mathbf{E}(\omega)$ to $\mathbf{j}(\omega)$ in the presence of damping (finite τ). Express your answer in terms of the dimensionless parameter $\omega\tau$.

(d) (*4 points*) Consider the two dimensional thin film geometry shown in the figure with a perpendicular magnetic field B_z. At equilibrium ($d\mathbf{p}(t)/dt = 0$) calculate the 2×2 resistivity matrix which relates the column vector containing the electric fields E_x and E_y to the corresponding column vector containing j_x and j_y. You will need to include the Lorentz force to answer this properly.

(e) (*2 points*) Find the conductivity matrix σ by inverting the resistivity matrix found in (d) above and show that σ obeys Onsager's relation $\sigma(-B)^T = \sigma(B)$ where T represents the transpose operation. If you do not remember the inversion formula, then solve the equations you found in part (d) for j_x and j_y in terms of E_x and E_y and write your answer in matrix form.