Student ID Number: ___________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part B, 4 January 2007, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

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B1. Consider a hydrogen atom immersed in a weak electric field $\mathbf{E} = \mathcal{E}\hat{\mathbf{z}}$. You may take the hamiltonian for this system to be

$$H = \frac{\vec{p}^2}{2m} - \frac{e^2}{4\pi|\vec{r}|} + e\mathcal{E}z \tag{1}$$

and you may regard the electron spin fixed with $m_s = +1/2$ throughout this problem. You are to find the energy shifts for the $n = 1, 2$ levels of hydrogen to first order in $\mathcal{E}$.

(a) (*3 points*) First recall the spectrum at zero field. Give the degeneracies of the $n = 1, 2$ levels, and the angular momentum quantum numbers l, m_l and parity $\pm$ of every state belonging to each of these energy levels.

(b) (*1 point*) Using a symmetry argument, explain why the first order shift of the $n = 1$ level is zero.

(c) (*2 points*) The zero field bound state wave functions are given by

$$\begin{aligned} \psi_{nlm}(\vec{r}) &= R_{nl}(r)Y_{lm}(\theta,\phi) \\ Y_{00} &= \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi} \\ R_{10} &= N_1 e^{-r/a}, \qquad R_{21} = N_2 r e^{-r/2a}, \qquad R_{20} = N_3(r-2a)e^{-r/2a} \end{aligned}$$

where the N_i are normalizing constants. Use the radial Schrodinger equation for $u \equiv rR$

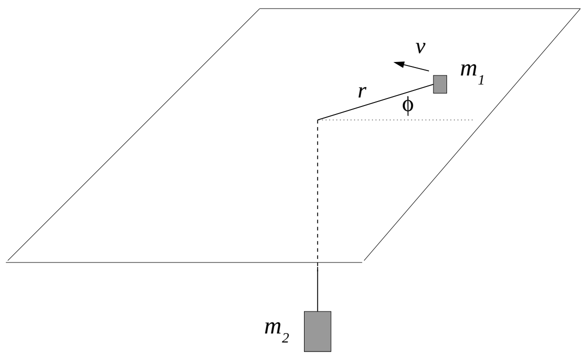
$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + \frac{\hbar^2 l(l+1)}{2mr^2} - \frac{e^2}{4\pi r}\right)u = Eu \tag{2}$$

to determine the parameter a in terms of $\hbar$, e and m.

(d) (*4 points*) Calculate the first order splittings of the $n = 2$ levels of hydrogen in a weak electric field. The zeroth order wave functions you will need are listed in part (c).

B2. One mass m_1 moves on a frictionless plane and is attached to a second mass m_2 via a string which goes through a hole in the plane as shown in the diagram. The radius r is the distance from m_1 to the hole. The mass m_2 is constrained to move only vertically. The acceleration of gravity is g.

For general motion, the angle ϕ is ignorable, and the angular momentum J, canonically conjugate to ϕ, is a constant of the motion.



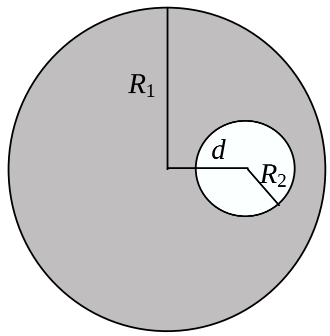
(a) (*2 points*) Find the angular velocity ω_0 and the angular momentum J for the special case that m_1 moves in a circle of constant radius r_0.

(b) (*3 points*) For a given J, find the general equation of motion for r. You do not need to solve this equation.

(c) (*3 points*) Assume that the motion of m_1 is nearly circular so that $r(t) = r_0 + \xi(t)$ with $\xi \ll r_0$. Find the frequency Ω (in radians/second) of the small radial oscillations. Give your answer in terms of m_1, m_2, r_0 and g.

(d) (*2 point*) From your previous answers, give the ratio Ω/ω_0 in terms of m_1, m_2, r_0 and g.

B3. An infinitely long cylindrical rod of radius R_1 is made of a material with a uniform electric charge density ρ per unit volume. A cylindrical cavity of radius R_2 exists within the confines of the material, with an axis parallel to but displaced from the axis of the rod by a distance d. Assume that $R_1 > (R_2 + d)$.



(a) (*5 points*) Determine the electric field everywhere within the cavity (of radius R_2). Please make sure to indicate what system of units you are using.

(b) (*5 points*) Determine the difference in the electric potential between the center of the rod and the center of the cavity.