Student ID Number: __________

PRELIMINARY EXAMINATION

Department of Physics
University of Florida
Part C, 5 January 2007, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may **NOT** use programmable calculators to store formulae.
2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.
3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.
4. Your assigned student **ID Number**, the **Problem Number**, and the **Page Number** should appear in the upper right hand corner of each sheet. Do **NOT** use your name anywhere on the Exam.
5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.
6. Each problem is worth 10 points.
7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*

DO NOT OPEN EXAM UNTIL INSTRUCTED

PRELIMINARY EXAMINATION

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C1. The energy of an harmonic oscillator in an eigenstate $|n\rangle$ of its Hamiltonian is:

$$\hat{H}|n\rangle = \hbar\Omega(n + \frac{1}{2})|n\rangle$$

In a thermal state $|\phi_{th}\rangle$ the n-th eigenstate of this harmonic oscillator is occupied with a probability

$$P_n = N \cdot e^{-n\beta} \qquad \beta \equiv \hbar\Omega/(k_B T)$$

(a) (*2 points*) Calculate the normalization constant N.

(b) (*4 points*) The average energy of the thermal harmonic oscillator is usually expressed as:

$$\bar{E}_{Thermal} = \hbar\Omega(n_{th} + \frac{1}{2})$$

Calculate n_{th} as a function of temperature T and eigenfrequency Ω.

(c) (*2 points*) Express the probability of the thermal harmonic oscillator to be in the n-th eigenstate as a function of n_{th}.

(d) (*2 points*) The density operator of the thermal state is:

$$\hat{\rho}_{th} = \sum_{n}^{\infty} P_n |n\rangle\langle n|$$

The difference between a mixed and a pure state stands out most clearly in a comparison between a thermal state and the thermal phase state:

$$|\phi_0\rangle \equiv \sum_{n=0}^{\infty} \sqrt{P_n}|n\rangle$$

The energy distribution of this state is identical to the energy distribution of the thermal state. Calculate the density operator

$$\hat{\rho}_{\phi} = |\phi_0\rangle\langle\phi_0|$$

of this state as a function of n_{th} and point out the main difference to the density operator of the thermal state.

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DEPARTMENT OF PHYSICS

UNIVERSITY OF FLORIDA

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C2. A self-contained chemical heating pad generates a constant heat output of $5\ W$ for $4\ hrs$ before the reactants are consumed and it begins to lose power. It is hermetically sealed inside a well insulated Styrofoam box of volume $1\ m^3$ containing air at a pressure of $1.0 \times 10^5\ Pa$ at an initial temperature of $20^o\ C$. The air has a specific heat capacity of $1020\ J/(kg \cdot K)$ and a density of $1.20\ kg/m^3$.

(a) (*4 points*) What is the temperature inside the box after $4\ hrs$ have passed?

(b) (*4 points*) What is the pressure in the box at that time?

(c) (*2 points*)) Is the system adiabatic? Why?

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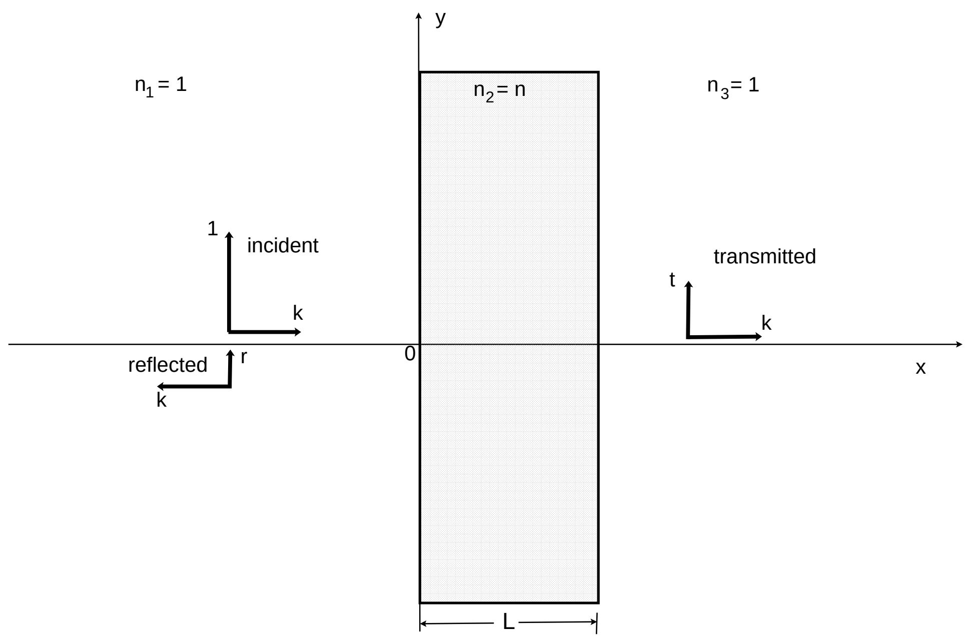
C3. The highest performing optical mirrors are made with dielectric coatings rather than metals. These coatings are characterized by a non-absorbing layer, with a real refractive index and near zero extinction coefficient. Here, you are to consider electromagnetic wave transmission and reflection by such materials.



Because there are no free carriers, and no magnetism, Maxwell's curl equations may be written (in cgs-Gaussian units) as

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{d\mathbf{D}}{dt} \qquad\qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{d\mathbf{H}}{dt},$$

where we have used $\mathbf{B} = \mathbf{H}$. The vectors $\mathbf{D}$ and $\mathbf{E}$ are related by

$$\mathbf{D} = \epsilon \mathbf{E} \equiv n^2 \mathbf{E},$$

where ϵ, the dielectric function, and n, the refractive index, are real materials constants. Take the electric field to be a plane wave

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\ \mathbf{r} - \omega t)},$$

where $\mathbf{E}_0$ is a (possibly complex) constant vector that defines the polarization and amplitude of the wave, $\mathbf{k} = k\hat{\mathbf{x}}$ is the wave vector, and ω the angular frequency of the wave.

(a) (*2 points*) Use the Maxwell equations above to show that $\mathbf{k}$, $\mathbf{E}$, and $\mathbf{H}$ form a mutually orthogonal right-hand set.

(b) (*2 points*) Show that the magnetic field and electric field amplitudes are related by $H = nE$ and that these two waves have the same phase.

(c) (*3 points*) Consider a wave normally incident on an interface between two media, 1 and 2. The wave is travelling in 1, along the x axis, polarized along y, and has unit electric field amplitude:

$$\mathbf{E}_{inc} = \widehat{\mathbf{y}} e^{i(kx-\omega t)},$$

The interface lies in the y, z plane, separating the two media with refractive indices n_1 and n_2. There will in general be a reflected wave, amplitude r, travelling along $-x$ in medium 1, and a transmitted wave, amplitude t, travelling along x in medium 2. Use the boundary conditions on the electromagnetic fields to calculate the quantities r and t.

(d) (*3 points*) Now consider a wave normally incident on a slab of thickness L. In this case, there are 2 plane interfaces. For simplicity take $n_1 = 1 = n_3$ and $n_2 = n$. The wave is incident in medium 1. Calculate the electric field amplitude in medium 3. You may express this *either* in terms of the reflected and/or transmission field amplitudes at the interfaces, k_2, and L, *or* in terms of n, k_2, and L.

Note. If $|x| < 1$, the sum $1 + x + x^2 \ldots$ is

$$\sum_{j=0}^{\infty} x^j = \frac{1}{1-x}.$$